

Semantics, languages and algorithms for multicore programming

Albert Cohen



Luc Maranget



Francesco Zappa Nardelli



Vote: topics for my next lecture

1. The lwarx and stwcx Power instructions 5

2. Hunting compiler concurrency bugs 6

- 3. Operational and axiomatic formalisation of x86-TSO
- 4. Fence optimisations for x86-TSO 3
- 5. The Java memory model 4

6. The C11/C++11 memory model 6

- 7. Static and dynamic techniques for data-race detection 4
- 8. What about the Linux kernel 2





1. Load Reserve / Store Conditional



RISC-friendly synchronisation operations

Load-reserve/Store-conditional

(aka LL/SC, larx/stcx and lwarx/stwcx, LDREX/STREX).

- can be used to implement CAS, atomic add, spinlocks, ...
- universal (like CAS) [Herlihy'93] (but no ABA problem)

Informally, stwcx succeeds only if no other write to the same address since last lwarx, setting a flag iff it succeeds.

Atomic Addition			
loop:	lwarx r, x;		
	add r,3,r;		
	stwcx r, x;		
	bne loop;		

What is no write since...

Informally, stwcx succeeds only if no other write to the same address since last lwarx, setting a flag iff it succeeds.

- In machine time?
 - (neither necessary, nor sufficient)
- Microarchitecturally: if cache-line ownership not lost since last lwarx

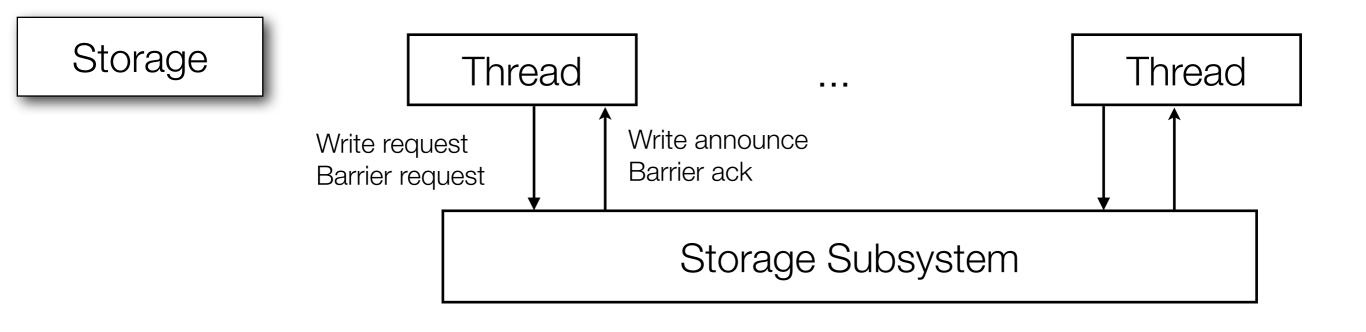
(simplified, and we don't want to model the microarchitecture)

Modeling no lost since

- Abstractly: ownership chain modeled by building up coherence order
- Coherence: order relating stores to the same location (eventually linear)

A stwcx succeeds only if it is (or at least, if it can become) coherence-next-to the write read from by Iwarx, and no other write can later come in between.

Isolate key concept: *write reaching coherence point* coherence is linear below this write, and no new edges will be added



The storage keeps: ...

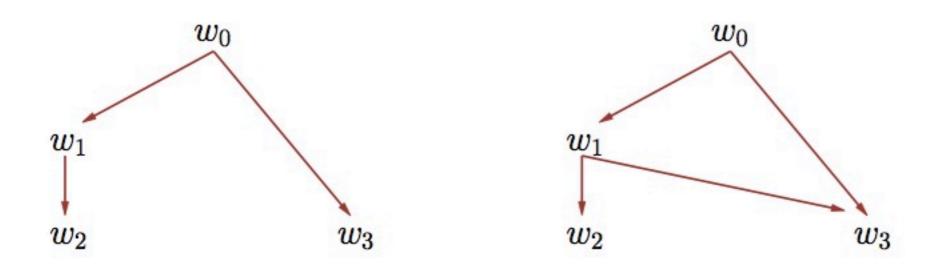
2. for each location, a partial order of coherence commitments

Idea 1: at the end of the execution, writes to each location are totally ordered. Idea 2: during computation, reads and propagation of writes must respect the coherence order (reduce non-determism of previous rules).

Intuition: if a thread executes x=1 and then x=2, another thread cannot first read 2 and then 1.

Coherence by Fiat

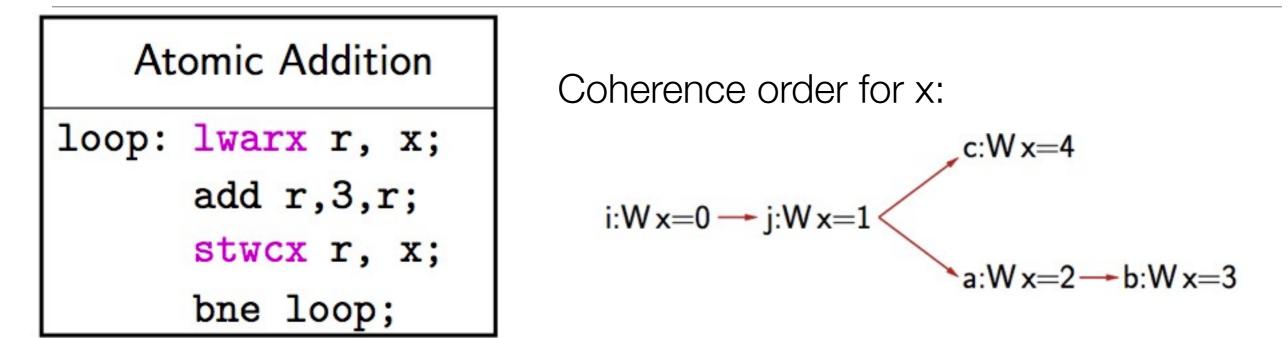
Suppose the storage subsystem has seen 4 writes to x:



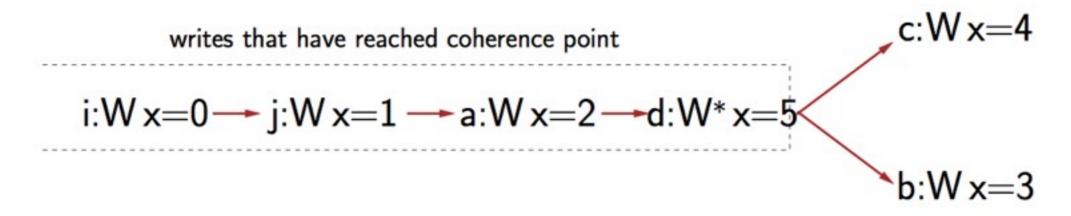
Suppose just $[w_1]$ has propagated to *tid* and then *tid* reads x.

- It cannot be sent w_0 , as w_0 is coherence-before the w_1 write that (because it is in the writes-propagated list) it might have read from;
- it could re-read from w₁, leaving the coherence constraint unchanged;
- It could be sent w_2 , again leaving the coherence constraint unchanged, in which case w_2 must be appended to the events propagated to *tid*; or
- It could be sent w_3 , again appending this to the events propagated to *tid*, which moreover entails committing to w_3 being coherence-after w_1 , as in the coherence constraint on the right above. Note that this still leaves the relative order of w_2 and w_3 unconstrained, so another thread could be sent w_2 then w_3 or (in a different run) the other way around (or indeed just one, or neither).

Coherence points and a successful stwcx



Suppose Iwarx reads from the a:W x=2. stwcx can succeed if this becomes possible:



Warning: stwcx can fail spuriously.

Load-reserve/store-conditional and ordering

- Same-thread load-reserve/store-conditionals ordered by program order;
- if all memory accesses are I-r/s-c sequences, then only SC behaviour;
- but normal loads/stores (to different address) not ordered; the l-r/s-c do not act as a barrier.

Confusion led to a Linux bug: bad barrier placement in atomic-add-return.

Synchronising C/C++ and POWER

Sarkar, Memarian, Owens, Batty, Sewell, Maranget, Alglave, Williams

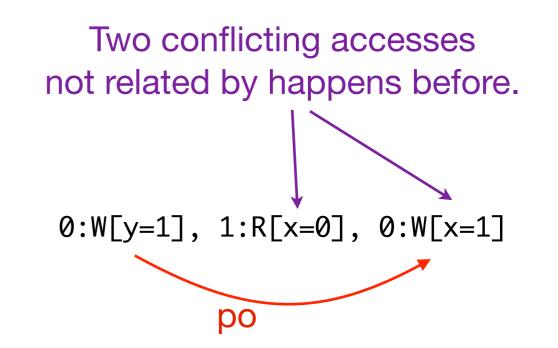
2. A word on techniques for data-race detection

Definition [data-race-freedom]: A program (traceset) is data-race free if none of its executions has two adjacent conflicting actions from different threads.

Equivalently, a program is data-race free if in all its executions all pairs of conflicting actions are ordered by happens-before.

Thread 0	Thread 1
*y = 1	if *x == 1
*x = 1	then print *y

A racy program



Recall: Happens-before

Definition [program order]: program order, <ppo, is a total order over the actions of the same thread in an interleaving.

Definition [synchronises with]: in an interleaving /, index i synchroniseswith index j, i $<_{sw}$ j, if i < j and A(I_i) = U (unlock), A(I_j) = L (lock).

Definition [happens-before]: Happens-before is the transitive closure of program order and synchronises with.

Examples of bonnons bofors			
Examples of happens before	Thread 0	Thread 1	
	*y = 1	lock();	
	lock();	<pre>tmp = *x;</pre>	
	*x = 1	unlock();	
	unlock();	if tmp = 1	
		then print *y	
hb			
hb 0:W[y=1], 0:L, 0:W[x=1], 0:U, 1: L, 1:R[x=1], 1:U, 1:R[y=1], 1:X(1) po po po po po po po po po po po hb SW 0:W[y=1], 1:L, 1:R[x=0], 1:U, 0:L, 0:W[x=1], 0:U			
po po po po	o po S(t	id) actions omitted.	

Data race detection: dynamic approaches

Modern high-performance dynamic race detectors are based either on:

happens-before ordering

lockset computation

reconstruct happens-before order in the current execution report a race if two conflicting accesses are not related by hb

no false positives

drawback: misses races occurring on rare executions

records which locks protect every memory access report a race if intersection of all locksets for a variable is empty

popularised by Eraser (Savage et al.) '97

can detect races not observed in the execution being monitored

drawback: unsound (false positives)

Examples of lockset computation

```
lock(b)
                   lock(a)
     lock(a)
                   x=2
     x=1
                   unlock(a)
     unlock(a)
           1:L(b);1:L(a);1:Wx1;1:U(a);2:L(a);2:Wx2;2:U(a)
locks held: 1:b
                 1:b,a
                                         2:a
C(x):
                              x:a,b
                                                     x:a
                lockset for x non-empty at the end, no data-race
     lock(b)
                   lock(c)
     lock(a)
                   x=2
     x=1
                   unlock(c)
     unlock(a)
       1:L(b);1:L(a);1:Wx1;1:U(a);2:L(c);2:Wx2;2:U(c)
                        x:a,b
                                              x:empty
C(x):
                  lockset for x empty at the end, possible data-race
```

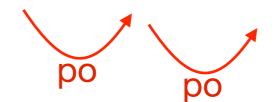
lockset vs happens-before



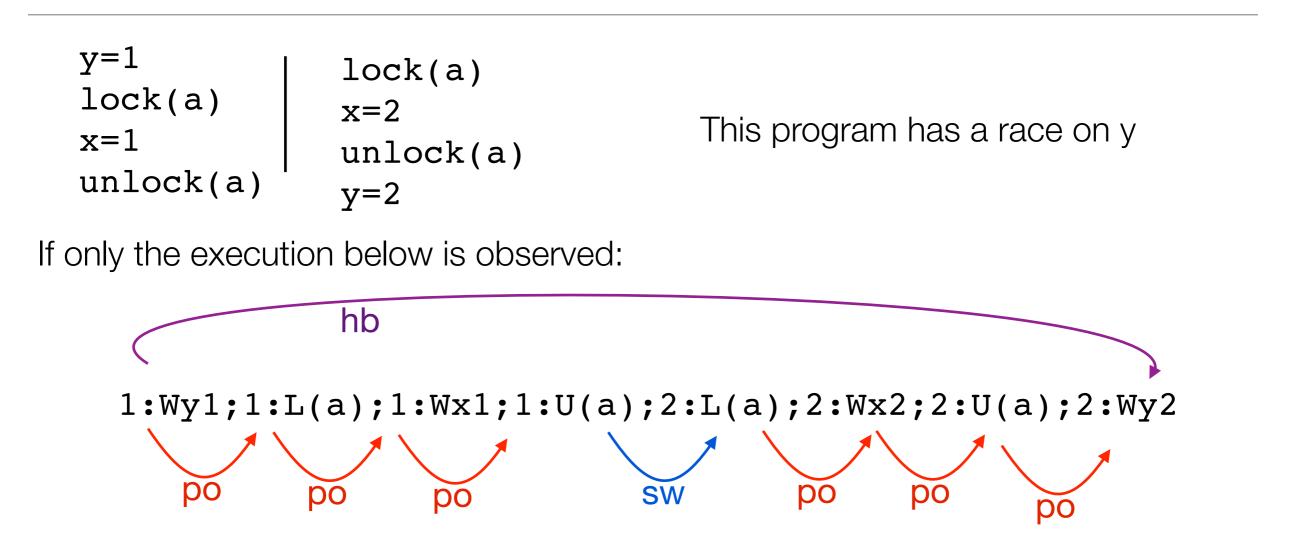
lockset vs happens-before

y=1	lock(a)
lock(a)	x=2
x=1	unlock(a)
unlock(a) '	y=2

This program has a race on y



lockset vs happens-before



happens-before computation does not report a race.

Lockset computation detects instead that accesses to y are unprotected and reports a possible race.

lockset vs happens-before (2)

y=1 I	lock(a)
lock(a)	tmp=x
x=1	unlock(a)
unlock(a)	if tmp == 1
	then print y

lockset vs happens-before (2)

y=1 I	lock(a)	
lock(a)	tmp=x	
x=1	unlock(a)	Th
unlock(a)	if tmp == 1	
	then print y	

This program instead is DRF.

lockset vs happens-before (2)

y=1 I	lock(a)		
lock(a)	tmp=x		
x=1	unlock(a)		
unlock(a)	if $tmp == 1$		
	then print y		

This program instead is DRF.

Happens-before computation will not report a race (no matter which execution is observed)

Since accesses to y are unprotected, locksets computation reports a false positive.

Data race detection

Modern high-performance dynamic race detectors are based either on:

happens-before ordering

reconstruct happens-before order in the current execution report a race if intersection if two conflicting accesses are not related by hb lockset computation

records which locks protect every memory access report a race if intersection of all locksets for a variable is empty

popularised by Eraser (Savage et al.) '97

sound

drawback: misses races occurring on rare executions

can detect races not observed in the execution being monitored

drawback: unsound (false positives)

Data race detection

M		h:
	Current state of the art:	L
rec	hybrid approaches combining locksets and happens-before ordering + other dynamic annotations	
rer con	Helgrind, RaceFuzzer, ThreadSanitizer	
	Impressive:	6-
	tolerable slowdown on large applications found thousands races	97
	Still not as reliable as the tool we dream of. Active area of research!	

Data race detection: static approaches

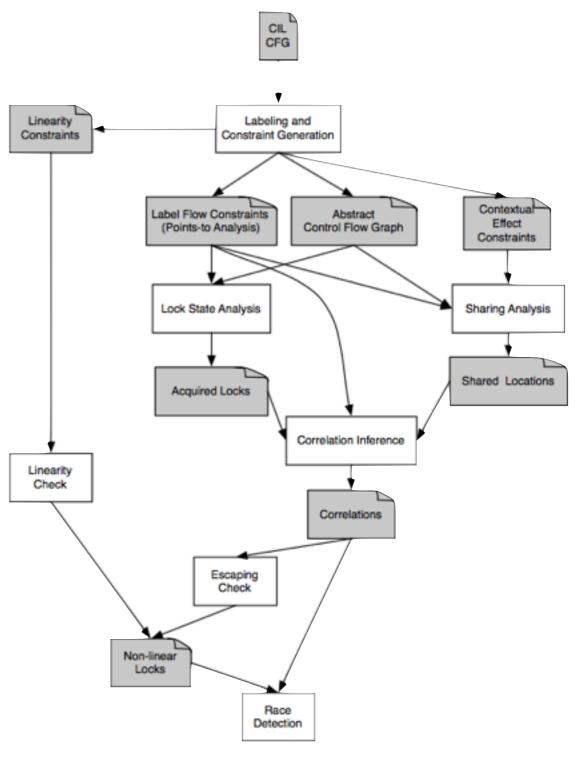
Run a bunch of static analysis for

inferring locksets.

Hard:

- aliasing on memory locations
- lock pointers
- must account all language features

Also done via fancy effect type-systems.





3. The C++11 memory model

a good example of an axiomatic memory model



The C++11 memory model

1300 page prose specification defined by the ISO.

The design is a detailed compromise: hardware/compiler implementability useful abstractions broad spectrum of programmers

Welcome to the official home of



2011-09-15: standards | projects | papers | mailings | internals | meetings | contacts

News 2011-09-11: The new C++ standard - C++11 - is published!

The syntactic divide

```
// for regular programmers:
atomic_int x = 0;
x.store(1);
y = x.load();
```

// for experts:

```
x.store(2, memory_order);
y = x.load(memory_order);
atomic_thread_fence(memory_order);
```

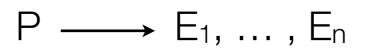
where *memory* order is one of the following:

mo_	_seq_	cst	mo_	release	mo_	_acquire
mo_	_acq_	rel	mo_	consume	mo_	relaxed

How may a program execute?

Two layer semantics:

1) a denotational semantics processes programs, identifying memory actions, and constructs candidate executions (*Eopsem*);



2) an axiomatic memory model judges *E*opsem paired with a memory ordering *X*witness

 $E_i \longrightarrow X_{i1}, \dots, X_{im}$

3) searches the consistent executions for races and uncostrained reads

is there an X_{ij} with a race?

Relations

An *E*_{opsem} part containing:

- sb sequenced before, program order
- asw additional synchronizes with, inter-thread ordering

An X_{witness} part containing:

- relates a write to any reads that take its value
- sc a total order over mo_seq_cst and mutex actions
- mo modification order, per location total order of writes

From these, compute synchronise-with (sw) and happens-before (hb).

We ignore *consume* atomics, which enables us to live in a simplified model. Full details in Batty et al., POPL 11.

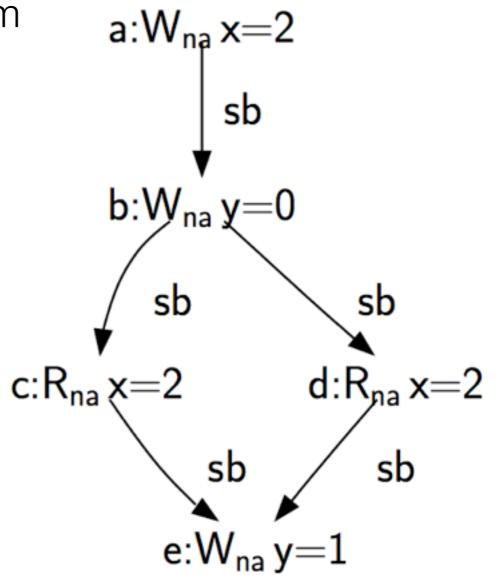
Formally

```
cpp memory model opsem (p : program) =
let pre executions =
  {(Eopsem, Xwitness). Opsem p Eopsem \wedge
     consistent execution (Eopsem, Xwitness) }
in
if \exists X \in \text{pre executions.}
    (indeterminate reads X = {}) V
    (unsequenced races X = \{\}) \vee
    (data races X = \{\})
then None
else Some pre executions
```

A single-threaded example

1. sequenced before (sb) - given by opsem

int main() {
 int x = 2;
 int y = 0;
 y = (x==x);
 return 0;
}



A single-threaded example

return 0;

}

1. sequenced before (sb) - given by opsem 2. read-from (rf) - part of the witness int main() { int x = 2; int y = 0; y = (x==x); W = 0 W = 2 W = 2 W = 2 V = 2 V = 2 S = 2S

rf

sb

Rx=2

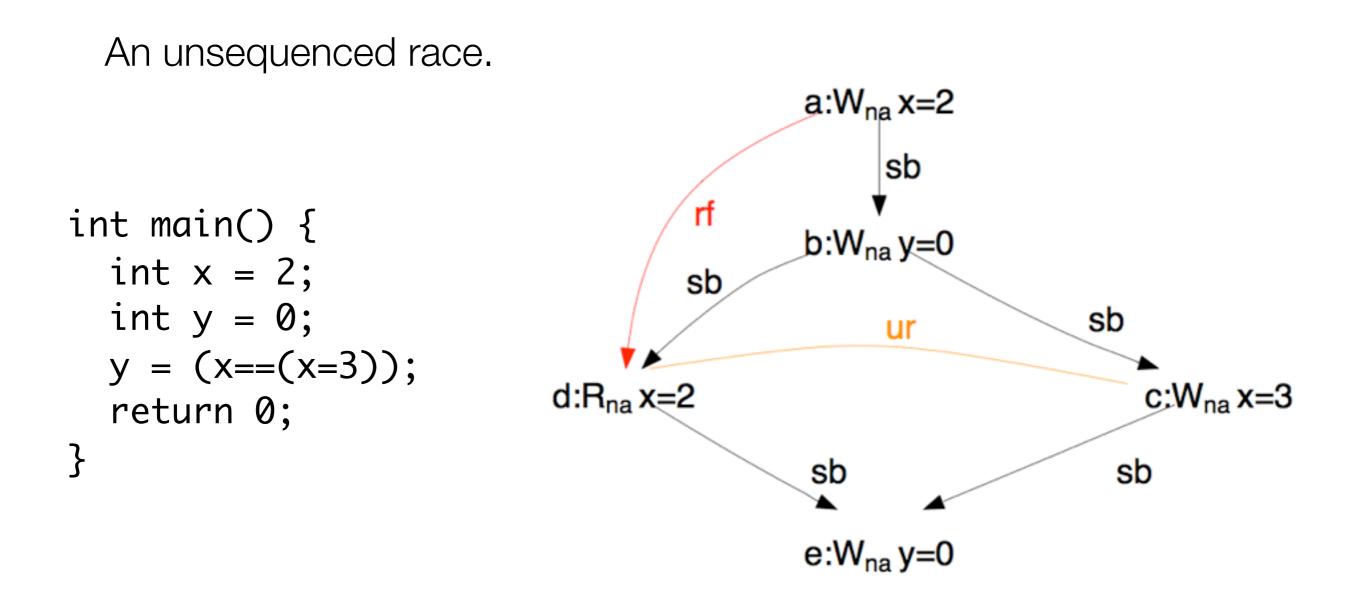
sb

Rx=2

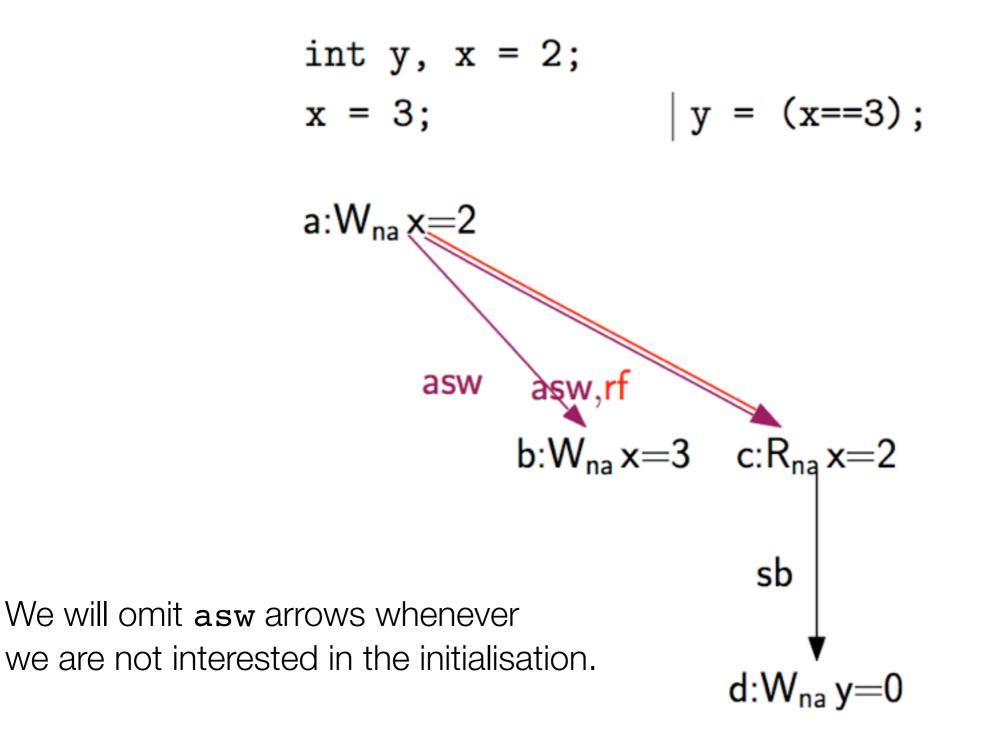
sb

 $W_{y=1}$

A single-threaded ex. with undefined behaviour



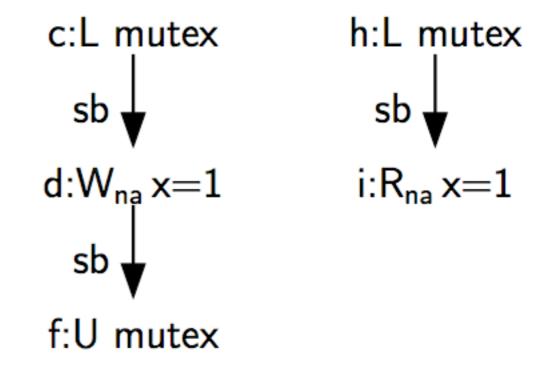
A simple concurrent program



Locks and unlocks

int x, r;	
mutex m;	
<pre>m.lock();</pre>	<pre>m.lock(); r = x;</pre>
$\mathbf{x} = \ldots$	r = x;
<pre>m.unlock();</pre>	

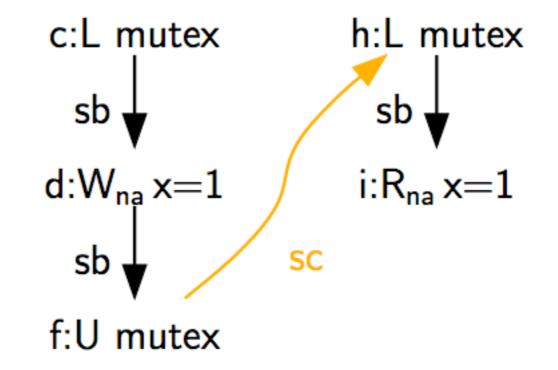
1. the operational semantics defines the sb arrows



Locks and unlocks

int x, r;	
mutex m;	
m.lock();	<pre>m.lock(); r = x;</pre>
x =	r = x;
<pre>m.unlock();</pre>	

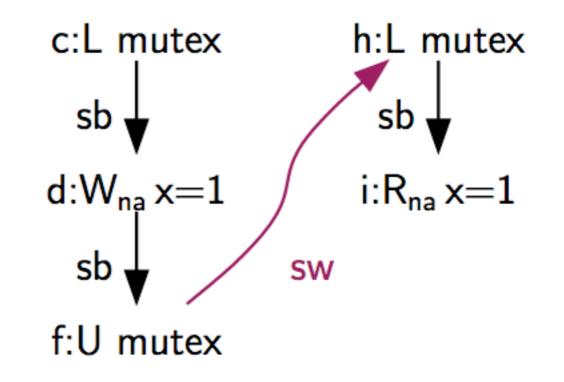
- 1. the operational semantics defines the sb arrows
- 2. guess an sc order on Unlock/Lock actions (part of the witness)



Locks and unlocks

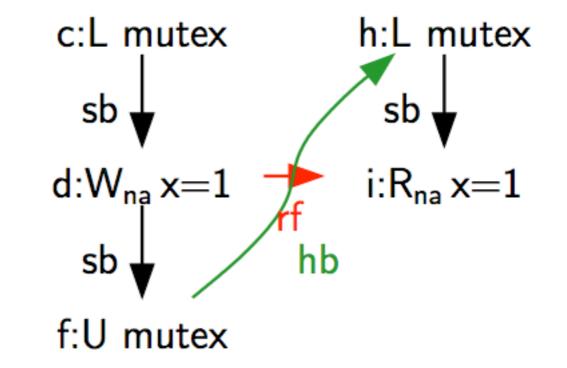
int x, r;	
mutex m;	
m.lock();	<pre>m.lock(); r = x;</pre>
x =	r = x;
<pre>m.unlock();</pre>	

- 1. the operational semantics defines the sb arrows
- 2. guess an sc order on Unlock/Lock actions (part of the witness)
- 3. the sc order is included in the syncronised-with relation



Locks and unlo	ocks	$\xrightarrow{simple-happens-before} =$
int x, r; mutex m;		$(\xrightarrow{sequenced-before} \cup \xrightarrow{synchronizes-with})^+$
m.lock();	m.lock()	;
<pre>x = m.unlock();</pre>	r = x;	

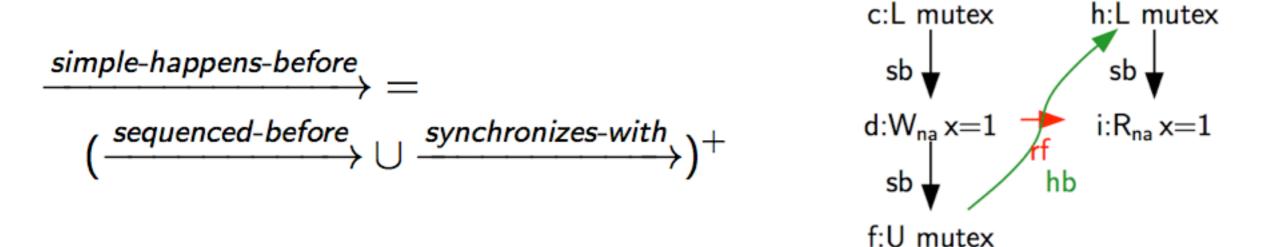
- 1. the operational semantics defines the sb arrows
- 2. guess an sc order on Unlock/Lock actions (part of the witness)
- 3. the sc order is included in the syncronised-with relation
- 4. which in turn defines the happens-before relation...



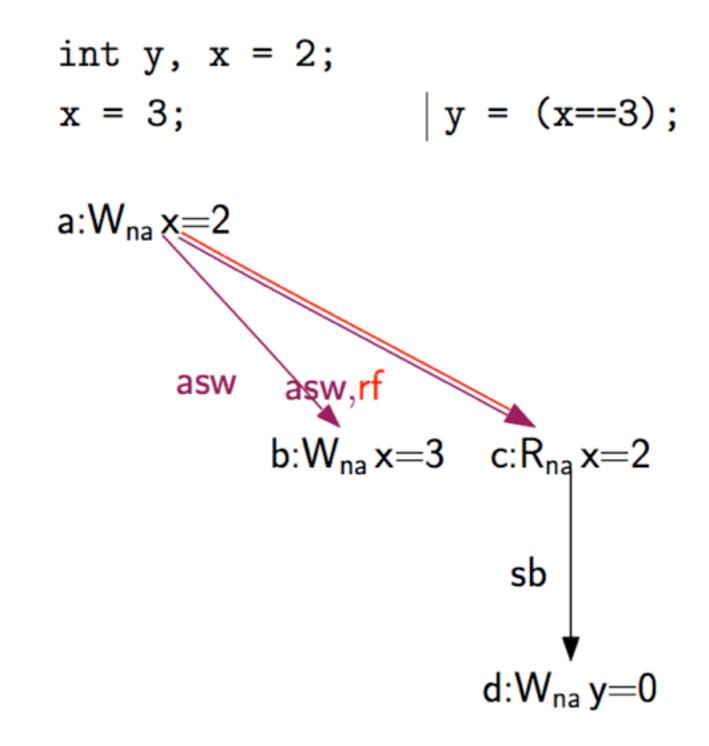
Happens before

The happens before relation is key to the model:

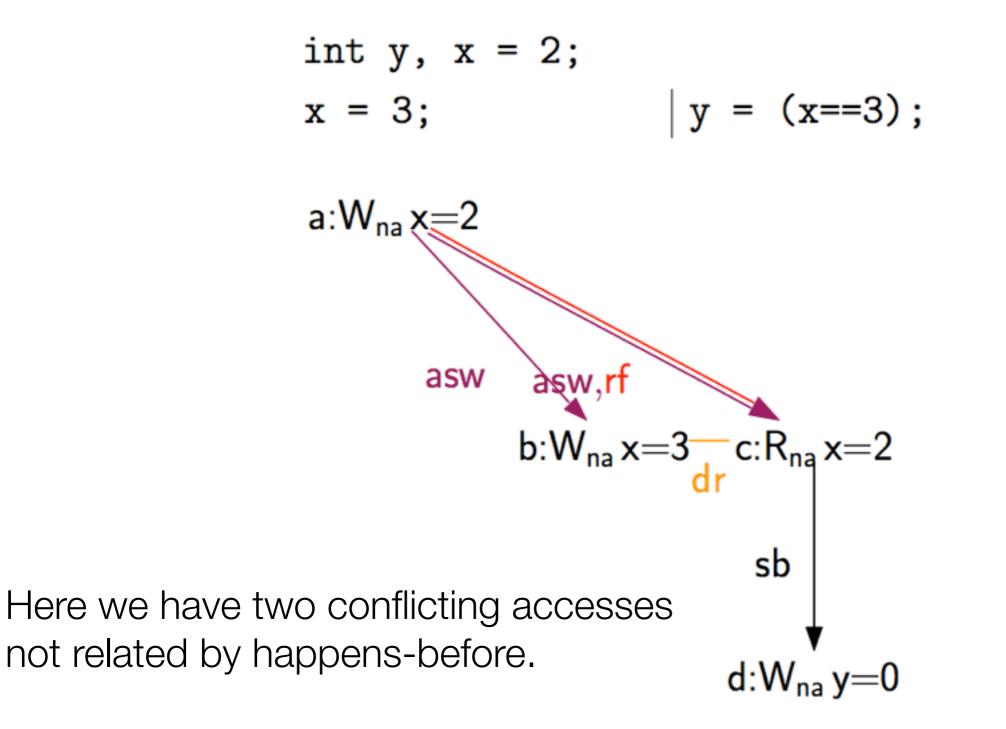
- non-atomic loads read the most recent write in happens before. (This is unique in DRF programs)
- 2. the story is more complex for atomics, as we shall see.
- 3. data races are defined as an absence of happens before between conflicting actions.



A data race



A data race



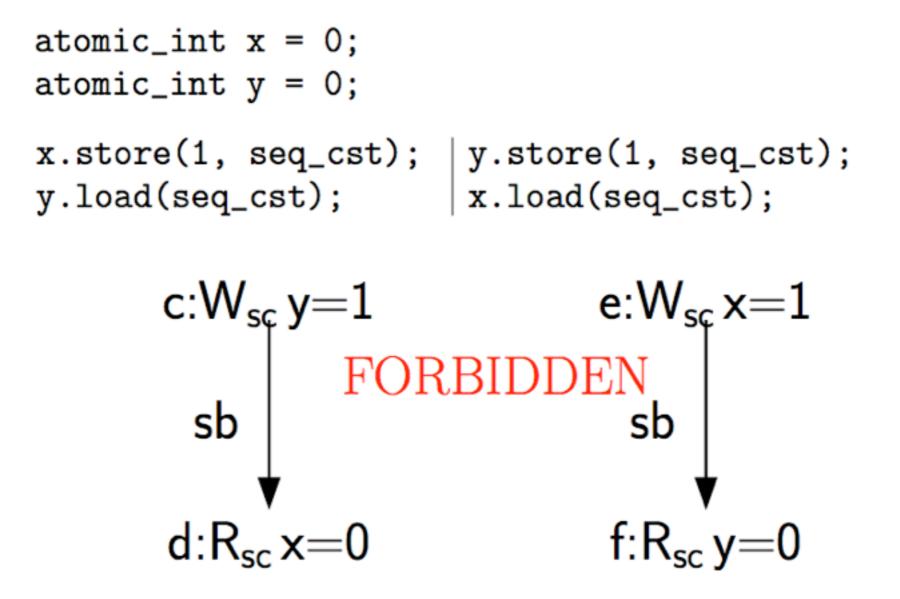
Data race definition

let data_races actions
$$hb =$$

{ (a, b) | $\forall a \in actions b \in actions$ |
 $\neg (a = b) \land$
same_location $a b \land$
(is_write $a \lor is_write b) \land$
 $\neg (same_thread a b) \land$
 $\neg (is_atomic_action a \land is_atomic_action b) \land$
 $\neg ((a, b) \in hb \lor (b, a) \in hb)$ }

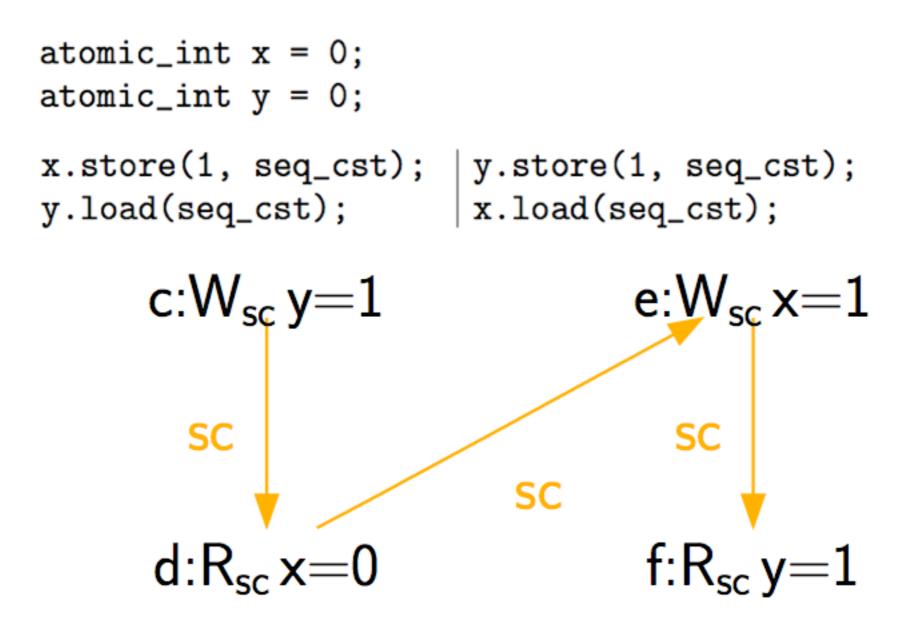
Programs with a data race have undefined behaviour (DRF model).

Simple concurrency: Dekker's example and SC



Why is this behaviour forbidden?

Simple concurrency, Dekker's example and SC



The sc relation must define a total order over unlocks/locks and seq_cst accesses... sc is included in hb, an rf must respect hb.

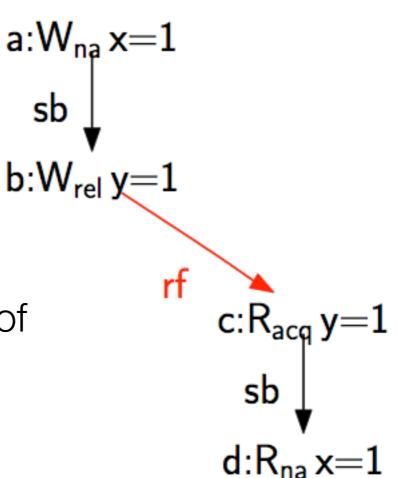
Expert concurrency: the release-acquire idiom

// sender

x = ...
y.store(1, release);

// receiver
while (0 == y.load(acquire));
r = x;

Here we have an **rf** arrow between a pair of release/acquire accesses.



Expert concurrency: the release-acquire idiom

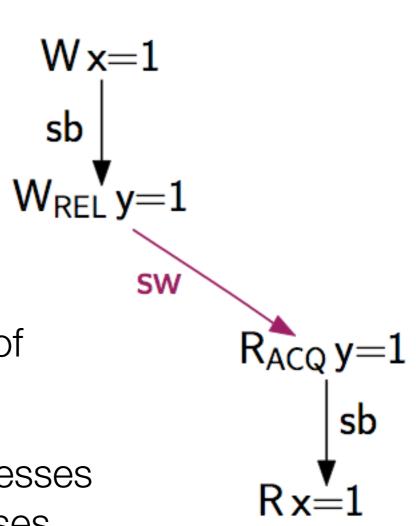
// sender

```
x = ...
y.store(1, release);
```

```
// receiver
while (0 == y.load(acquire));
r = x;
```

Here we have an **rf** arrow between a pair of release/acquire accesses.

The rf arrow between release/acquire accesses induces an sw arrow between those accesses.



Expert concurrency: the release-acquire idiom

// sender

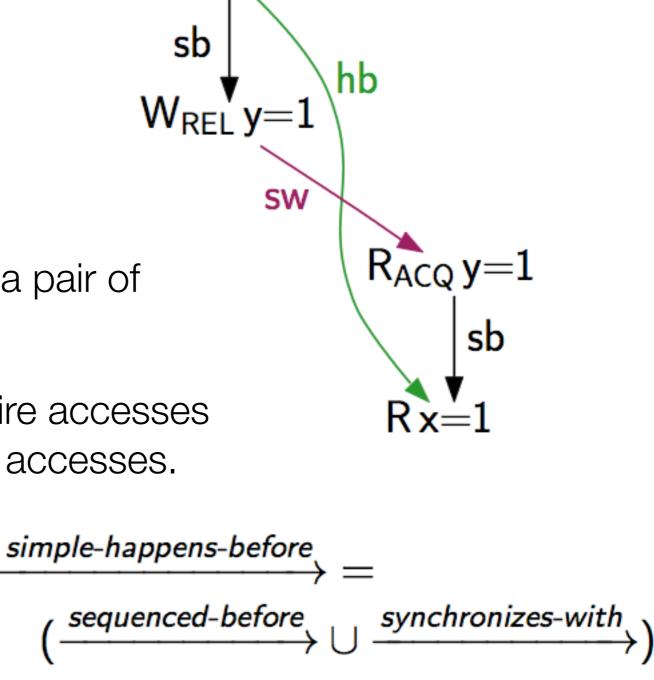
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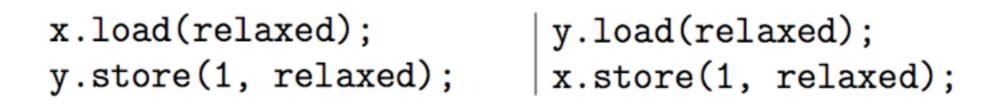
The rf arrow between release/acquire accesses induces an sw arrow between those accesses.

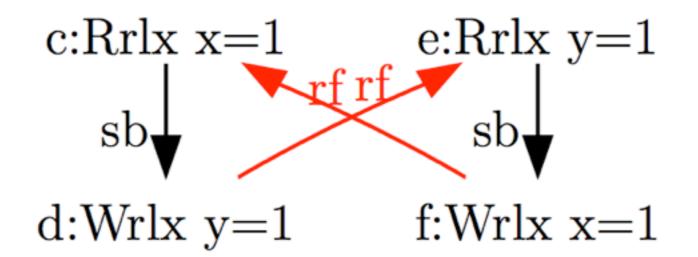
And in turn defines an hb constraint.



Wx=1

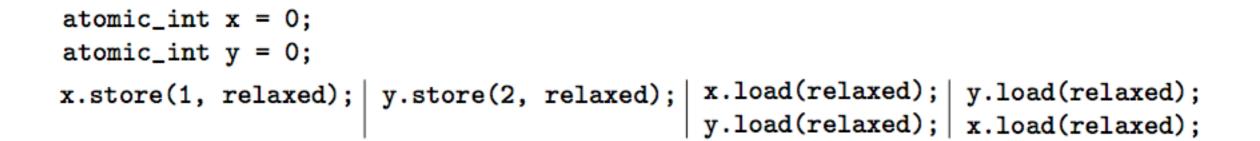
Relaxed writes

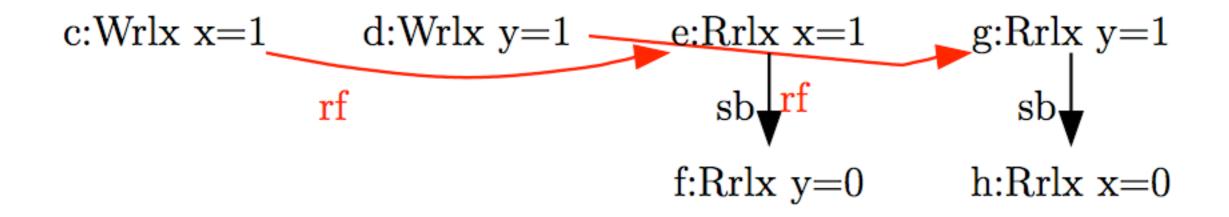




No data-races, no synchronisation cost, but weakly ordered.

Relaxed writes, ctd.





Again, no data-races, no synchronisation cost, but weakly ordered (IRIW).

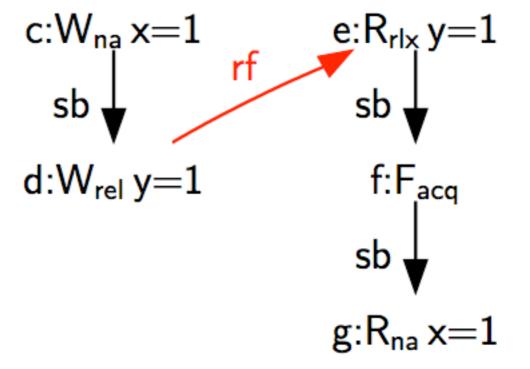
// sender	// receiver
x =	<pre>while (0 == y.load(acquire));</pre>
<pre>y.store(1, release);</pre>	r = x;

```
// sender
x = ...
y.store(1, release);
// receiver
while (0 == y.load(relaxed));
fence(acquire);
r = x;
```

```
// sender
x = ...
y.store(1, release);
```

// receiver
while (0 == y.load(relaxed));
fence(acquire);
r = x;

Here we have an **rf** arrow between a release write and a relaxed write.

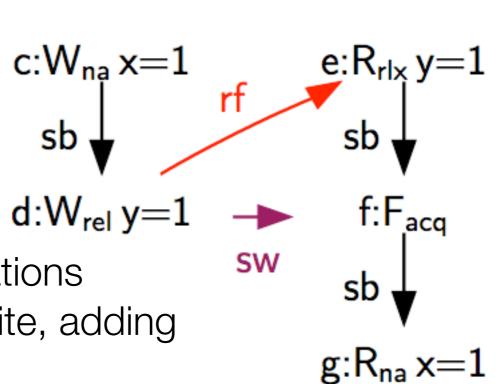


```
// sender
x = ...
y.store(1, release);
```

// receiver
while (0 == y.load(relaxed));
fence(acquire);
r = x;

Here we have an **rf** arrow between a release write and a relaxed write.

The acquire fence follows the sb/rf relations looking for the corresponding release write, adding a sw arrow.



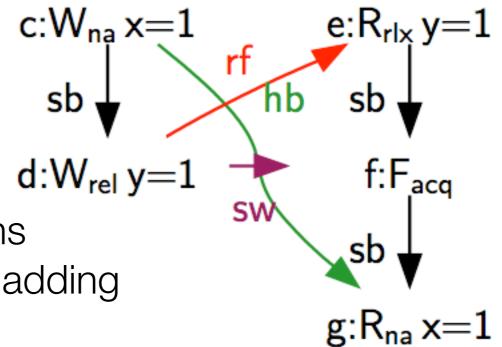
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r = x;

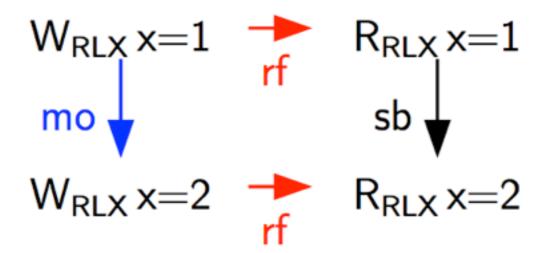
Here we have an **rf** arrow between a release write and a relaxed write.

The acquire fence follows the sb/rf relations looking for the corresponding release write, adding a sw arrow.

Happens-before follows as usual...



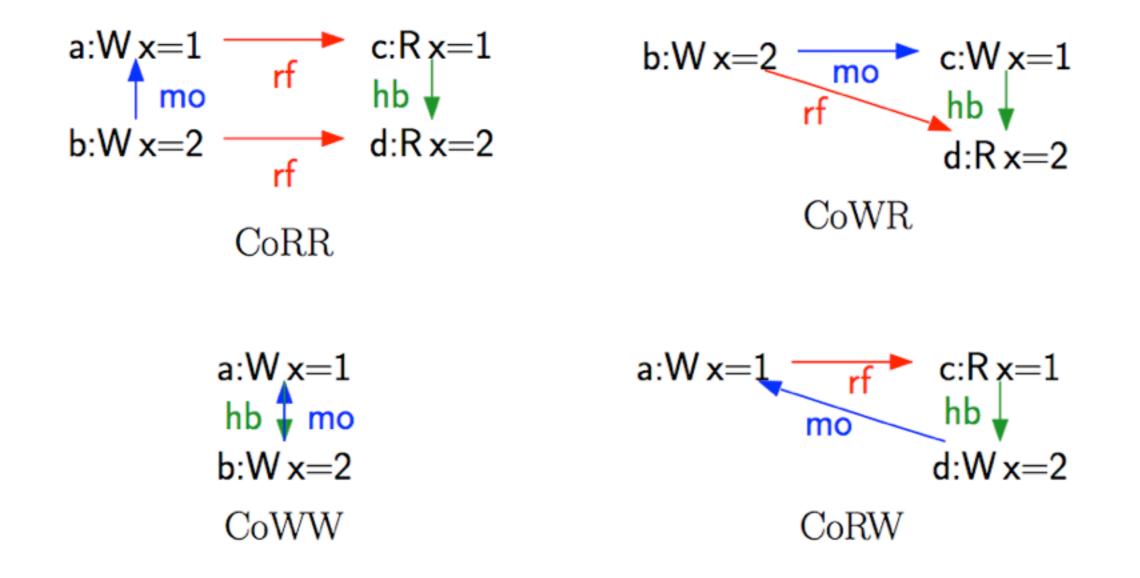
Modification order (aka coherence)



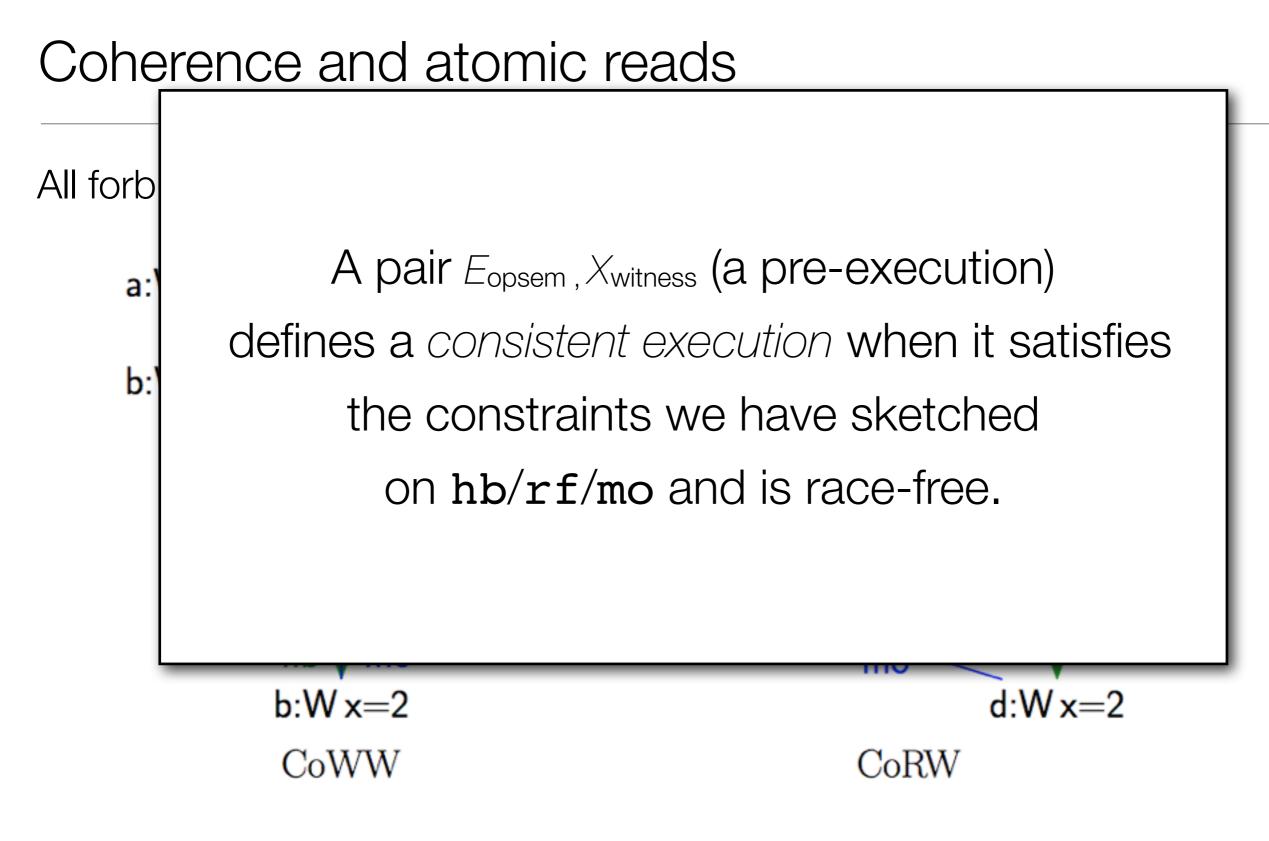
Modification order is a total order over atomic writes of any memory order.

Coherence and atomic reads

All forbidden:



Idea: atomics cannot read from later writes in happens-before.



Idea: atomics cannot read from later writes in happens-before.

The full model

$a \xrightarrow{r} b = (a, b) \in r$	is store a = case a of STORE $\to T \parallel _ \to F$		
			visible_side_effect_set actions threads location-kind sequenced before additional-genchronized with data-dependency toppens before =
$a r b = (a, b) \in r$	is dence $\mathfrak{z} = case \ \mathfrak{z} \text{ of } FENCE ___ \to T \parallel_ \to F$	ru_element st_bead a = same_filteend a st_bead V is_atomic_runv a	$\{a \in happens before the (a, b) = ab in visible_side_effects actions for additional synchronized with data-dependency control-dependency happens before a b)$
$a \stackrel{s}{\neq} b = (a, b) \notin r$	is_lock_or_unlock a = is_lock a V is_unlock a	release-sequence $= \lambda_{id} \xrightarrow{intare sequence} b =$	viable_express_i_site_effect_uill = viable_express_i_site_effect_uill var_best b =
	is_atomic_action a = is_atomic_load a ∨ is_atomic_store a ∨ is_atomic_rmw a	is_attammLocation $b \land$ is_reference $s_{ad} \land ($ $(b = a_{ad}) \lor$	(c.vase,head additionization, c.n. -(b. <u>summahar</u>) c.). ('a. vase,head additionization, a multitationation, c.
$\stackrel{r}{\longrightarrow} = r$	is_load_or_store a = is_load a V is_store a	$(n_{k}\text{-lement} a_{k} \land h_{k} \text{-mainfature-only} b.)$ $((C_{k} a_{k} - \frac{\text{mainfature-only}}{(m \in A_{k})} b \implies mainfature (h))$	
$a \xrightarrow{c} b \xrightarrow{s} c = a \xrightarrow{c} b \wedge b \xrightarrow{s} c$			myimage $f s = \{y, \exists x \in s, (y = f x)\}$
relation_over $s \ rel = {\rm domain} \ rel \subseteq \ s \wedge {\rm range} \ rel \subseteq \ s$	is_ntomic_load a∨is_atomic_rmw a∨is_load a	release_sequence_set actions threads location-kind sequenced before additional-synchronized-with data-dependency control-dependency modification-order = release_sequence_actions threads location-kind sequenced before additional-synchronized-with data-dependency control-dependency modification-order a b)	risible_exquences_al_side_stfects = risible_exquences_al_side_stfects =
$\xrightarrow{nt} _{s} = rel \cap (s \times s)$	is_write a = is_utomic_store a∨is_utomic_rmw a∨is_store a		$\chi(vex.bad, b)$. (b,if) is Lut _stemic_bostion b then (vex.bad) U
$rel _s = rel \cap (s \times s)$		hypothetical_release_sequence = $2 \frac{h_{partitescal-induce sequence}}{h_{partitescal}} b = h_{partitescal-induce sequence} b = (b = a) \vee (b = a) $	visilie_copense_sf_side_sflects_tail vane_lead b ebc {}
$\frac{nd}{n} _s = nd \cap (s \times s)$	is_acquire a = (case memory_order a of SOME mem_ord →	$(r_{x}chment \ b \ h \ a \ mathematical sets \ b \ h \ (v_{x}, a \ mathematical sets \ b \ h \ mathematical sets \ b \ h \ mathematical sets \ b \ mathematical sets \ sets \$	vielle_sequences_of_side_stflects_set actions threaks location-kind sequenced-before additional synchronized-with data-dependency control-dependency modification-order happens-before visible-side-effect =
$rel _s = rel \cap (s \times s)$	(mem.cod ∈ (Mo_ACQUIRE, Mo_ACQ_REL, Mo_SEQ_CST } ∧ (\$z.read a ∨ is_fence -)) ∨ (* 29 a8 5 states that consume fences are acquire fences. *)		wjimage (telika wypenenu, d. akir ufferts actions through location-kind sequences before additional quedrosined with data dependency control-dependency modification order happens before valide side effect
strict_preorder ord = irreflexive ord \ trans ord	(* 29.85 states that consume fences are acquire fences. *) ((mem_ord = MO_CONSUME) ∧ is_fence a) NONE → is_lock a)	hypothetical_release_sequence_set actions threads location-kind sequenced-before additional-synchronized-with data-dependency control-dependency modification-order = hypothetical_release_sequence actions threads location-kind sequenced-before additional-synchronized-with data-dependency control-dependency modification-order a b)	consistent_read_from_mapping = consistent_reads_from_mapping = (rbt, (siz-read b \ is_it_con_it_consistent_reads_from_mapping) = (rbt, (siz-reads_from_it_consistent_reads_from_mapping) = (rbt, (siz-reads_from_it_consistent_reads_from_it_cons_stent_reads_from_it_cons_stent_reads_from_it_cons_stent_reads_from_it_cons_stent_reads_from_it_cons_stent_reads_from_it_cons_stent
	is.consume a =	synchronizes, with $a = 2 \xrightarrow{performers} b =$	$\begin{array}{l} (10, 1 - 1 - 1) \rightarrow \\ (10, 1) - 1 - 1) \rightarrow \\ (10, 1) - 1) - 1 -$
total_over s ord = relation_over s ord \land $(\forall x \in s. \forall y \in s. x \xrightarrow{ad} y \lor y \xrightarrow{ad} x \lor (x = y))$	is_read a ^ (memory_order a = SOME MO_CONSUME)	$\binom{*}{2}$ - additional synchronization, from thread create etc *) $\frac{2}{2}$ - $\frac{1}{2}$ - $\frac{1}{2}$ + $\frac{1}{2$	else $-(\exists z, z \rightarrow b)) \land$ (v?b. (ix_rreal $b \land is_{z,t}$ atomic_location $b) \implies$
	is_release a = (case memory_order a of	(sume_location ≥ b ∧ ≥ ∈ zctions ∧ b ∈ zctions ∧ ((* - mutex spectronizzion - *)	$(\mathbf{ff} = [\exists b', vas] \in viable acquarters of side effects: (b' = b))then (\exists b', vas) \in visble sequences of side effects: (b' = b) \cap \{\exists c \in uss : c = b\}$
<pre>strict_total_order_over s and = strict_preorder and \Lambda total_over s and</pre>	Some mem_and \rightarrow mem_and $\in \{MO, RELEASE, MO_ACQ_REL, MO_SEQ_CST\} \land$ (is, write a \vee is, fence a)	$(i_{ij}, milock > A \ i_{ij}, lock < A A = \stackrel{S}{\rightarrow} b) \vee$ (* - release, locquire synchronization = *)	$e_{\text{the }} = (\Xi_{n,2} \neq \phi_{j})_{j} \wedge e_{\text{the }} = (\Xi_{n,2} \neq \phi_{j})_{j}$
$x \xrightarrow{acd}_{pred} y =$ $pred \times \wedge x \xrightarrow{acd}_{y} y \wedge -(\exists z. pred z \wedge x \xrightarrow{acd}_{y} z \xrightarrow{acd}_{y} y)$	\parallel NONE \rightarrow is_unlock a)	(is release $\lambda \land is acquire b \land \neg same thread z \land \land(3c. z \xrightarrow{minus sequence} c \xrightarrow{d} b)) \lor$	((n)∈
	is_seq_est a = (memory_order a = SOME MO_SEQ_CST)	(* – fence synchronization – *) (is_fence a ∧ is_refease = a ∧ is_ence b ∧ is_acquire b ∧	$\sum_{a} \frac{d_{anisove bidw}}{d_{anisove bidw}} b \wedge \\ \text{same_location } a b \wedge h_{a,a,a,d,comic_allocation } b \\ = \omega (x = y) \sqrt{x} \frac{d_{a,a}(x = y) + x}{d_{a,a}(x = y) + x} \frac{d_{a,a}(x = y)}{d_{a,a}(x = y)} d_{a,$
$\begin{array}{c} x \xrightarrow{ood} y = \\ x \xrightarrow{ood} y \wedge \neg (\exists x. x \xrightarrow{ood} z \xrightarrow{ood} y) \end{array}$	location_kind = MUTEX	$(\exists c. 2\gamma, same location x y \land$ $ s. totics into x \land h a station x \land h a station x \land h a station x \land h a subsection that x \land y as superscription the to X \land$	(* anc caller, *) // / / / / / / / / / / / / / / / / /
well_founded $r = wf r$	NON_ATOMIC ATOMIC	(∃x, x ³ pathoticit - datase sequence), z = -5, y))) ∨	$\forall c \in c$
type_abbrev_action_id:string	actions_respect_location_kinds =	(is_frace ≥ Λ is_release ⇒ Λ is_atomic_action b / is_accupite b ∧ [25: sum_beating x b ∧	$\Rightarrow (c = a) \lor c = a$
	∀a. case location a of SOME 1 → (case location kind 1 of MUTEX → \$LobeLor enables a	is_atomic_action x ∧ is_write x ∧ 2 meansofabley x ∧	$(V(\mathbf{A}) \in \frac{\operatorname{derm} Adap}{V}, V \in V$
type_abbrev thread_id : string	$NON_ATOMIC \rightarrow is_load_or_store a$ $ATOMIC \rightarrow is_load_or_store a \lor is_atomic_action a$	$(\exists x, x \text{functions} \text{-actions} x \neq (-b))) \lor$ (is_atomic_action $x \land \text{is_referes } x \land$	$c \xrightarrow{-} a \land$ is write $b \land same bostion a b \land is at atomic-bostion a \Rightarrow c \xrightarrow{-} c \xrightarrow{-} c \xrightarrow{-} b \land is at atomic-bostion a$
type_abbrev location : string	NONE → T	is_dence b∧is_acceptive b∧ (2): sume_location x ∧ h summary b∧ summary b∧ (2): summary b∧ (2): summar	(Y(a, b) ∈ ^{dd} , is_atomic_mw b
type_nbbrev val:string	is_at_location_kind = is_at_location_kind = case location a of	$\begin{array}{c} x & \longrightarrow & 0 \\ (\exists z, z \xrightarrow{d \text{ substance sequence }} z \xrightarrow{d'} x))))) \end{array}$	$\Rightarrow 2 \xrightarrow{\text{contractive over}} b) \land \land \land (\gamma(x, b) \in \overset{d}{\prec}, \text{ is,seq.est } b$
memory_order_enum = MO_SEQ_CST	Some $l \rightarrow (location-kind l = lk0)$ $\parallel NONE \rightarrow F$	synchronizes_with_set actions threads location-kind sequenced-before additional-genchronized with data-dependency control-dependency of modification-order sc release-sequence hypothetical-release-sequence =	$ = \left(\frac{1}{1 + 1} \left(\frac{1}{1 + 1} \right) + \frac{1}{1 + 1} \left(\frac{1}{1 + 1} \left(\frac{1}{1 + 1} \right) + \frac{1}{1 + 1} \left(\frac{1}{1 + 1} \right) + \frac{1}{1 + 1} \right) \right) $
MO_RELAXED MO_RELEASE MO_ACQUIRE	is_at_mutex_location s = is_at_location_kind z MUTEX	synchronizes, with actions through sociation-kind sequenced-before additional-synchronized-with data-dependency control-dependency of modification-order sc release-sequence hypothetical-release-sequence a b)	(* free realizion *)
MO_CONSUME MO_ACQ_REL		extrise_u_dependency_Lo = $a^{\text{contex}+dependency}_{\text{int}} b =$ $a^{(d)}_{\text{int}} = \frac{a^{\text{contex}+dependency}_{\text{int}}}{a^{(d)}_{\text{int}}} b^{(d)}_{\text{int}} = b^{(d)}_{\text{int}} + b^{$	(* 23.3.*) (*2.*)(2.5) =
action =	is_nt_non_ntomic_location = is_nt_location_kind = Non_ATOMIC		$(i_k \text{ force } \times \Lambda \hat{i}_k \text{ support } \Lambda \times \Lambda \hat{i}_k \text{ summization } b \land$ $i_k \text{ write } a \land \text{ max}, \text{ location } a b \land$ $a \stackrel{c}{\to} x \land n \stackrel{c}{\to} b)$
LOCK of action,id thread.id location UNLOCK of action,id thread.id location ATOMIC_LOAN of action,id thread.id memory_order_enum location val	is_at_atomic_location a = is_at_location_kind a ATOMIC	carries_u_dependency_to_set actions threads location-kind sequenced-before additional-synchronized-with data-dependency control-dependency if = curries_u_dependency_to_actions threads location-kind sequenced-before additional-synchronized-with data-dependency control-dependency if a b}	$\longrightarrow (y - z) \lor z \xrightarrow{\text{deficition odd}} y) \land$ (* 29.34 *)
ATOMIC_STORE of action.jd thread_id memory_order_enum location val ATOMIC_LIMW of action.jd thread_id memory_order_enum location val val LOAD of action.jd thread.jd location val	same_thread $ab = (\text{thread}_{id}, of a = \text{thread}_{id}, of b)$		(V(x,x)) ⊂ = maximizing , V(x,y) ∈ <i>d</i> , (Castania estimation > h.estres = h.estres = x ∧
STORE of action_id thread_id location val FENCE of action_id thread_id memory_order_enum	same_thread a b = (thread_id_ot a = thread_id_ot b)	a ∈ actions ∧ d ∈ actions ∧ (∃b, is_release a λ is_consume b ∧	is surfice 2 \wedge same, bottime a $b \wedge x^{-1}$ $x \leq b \wedge (a, starting a + b + a)$ $\Rightarrow (y = 2) \vee 2^{-\frac{2\pi + b}{2} + 1} + \frac{2\pi + b}{2} \vee 2^{-1} + 2\pi +$
(action_id_of (LOCK aid) = aid) ∧	threadwise_relation_over $s \operatorname{rel} =$ relation_over $s \operatorname{rel} \land (\forall (a, b) \in \operatorname{rel}. \operatorname{same_thread} a b)$	$(\exists c. 2 \xrightarrow{ntime superface} c^{-1}, b) \land$ $(b \xrightarrow{contex-superface} c_{-1}, d) \land$	(* 2035 *)
$(action_id_of (UNLOCK aid _) = aid) \land$			
(action_id_of (ATOMIC_LOAD aid) = aid) \land (action_id_of (ATOMIC_STORE aid) = aid) \land	same_location $z = b = (bcation \ z = bcation \ b)$	dependency_ordered_before_set actions threads location-kind sequenced-before additional-synchronized-with data-dependency control-dependency of modification-order release-sequence carries-a-dependency-to =	$(Y(\mathbf{x}, \mathbf{x}) \in \frac{\text{inparent states}}{1}, Y(\mathbf{y}, \mathbf{x}) \in \frac{\text{inparent states}}{1}, Yz.$ $(\mathbf{x}_1, \mathbf{x})$ there is $\lambda \in \mathcal{N}$ is $\lambda \in \mathcal{N}$. It is $\lambda \in \mathcal{N}$ is $\lambda \in \mathcal{N}$.
$(action.id.u.d. (ATOMIC_LOAD aid) = aid) \land$ $(action.id.u.d. (ATOMIC_STORE aid) = aid) \land$ $(action.id.u.d. (ATOMIC_BINW aid) = aid) \land$ $(action.id.u.d. (ATOMIC_BINW aid) = aid) \land$ $(action.id.u.d. (STORE aid) = aid) \land$	same_location $z = (location z = location b)$ locations_of actions = [I. Eq. (location z = SOME I])	dependency_undered_before_set actions threads location-kind sequenced before additional-spectrosized with data dependency control dependency of modification-order release sequence carries a dependency-to a - dependency_undered_before_actions_threads_location-kind sequenced before additional-spectrosized with data dependency control dependency of modification-order release sequence carries a dependency to a 8)	$[b_{int} contact = A \land b_{int} contact A \land b$
$(action.id.of (ATOMIC ALGOAD aid) = aid) \land$ $(action.id.of (ATOMIC STORE aid) = aid) \land$ $(action.id.of (ATOMIC MANN aid) = aid) \land$ $(action.id.of (LOAD aid) = Aid) \land$	locations of actions = { <i>l</i> . 3a. (location a = SOME <i>l</i>)}	dependency undered_before actions threads location-kind aspanende before additional-synchronized with data-dependency control-dependency of modification-order release-aspanence corrito-a-dependency-to-a-b}	$ \begin{array}{l} [b_{ad} \min_{x \in A} t_{bd} \max_{x \in A} t_{bd} \max_{x \in A} t_{x} \wedge t_{bd} \max_{x \in A} t_{x} \wedge t_{bd} + t_{bd} \max_{x \in A} t_{bd} + t$
$\begin{array}{l} \left(\operatorname{arcinulation}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \right) \\ \left(\operatorname{arcinulation}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arcinulation}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arcinulation}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arcinulation}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arcinulation}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arcinulation}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arcinulation}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arcinulation}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \\ \left(\operatorname{arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \\ \left(\operatorname{arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \\ \left(\operatorname{arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \\ \left(\operatorname{arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \\ \left(\operatorname{arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \\ \left(\operatorname{arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \\ \left(\operatorname{arconstructure}_{\mathcal{A}} \left(\operatorname{Arconstructure}_{\mathcal{A}} \right) \right) \right) \\ \left(\operatorname{arconstructure}_{$	locations_d actions = (t ≥ 2a (location a = 5000 f)) well_formed_action a = case a significant actions = memory = − = memory = ∈	dependency_unitered_before actions threads location kind sequenced before additional-gendennized with data-dependency costrol-dependency of modification-order relaxes sequence carries-a-dependency-to a b) simple_lappent_before = simple_lappent_be	$[b_{int} contact = A \land b_{int} contact = A$
$ \begin{array}{l} \left(\operatorname{ariseshaf}_{(n)} \left(\operatorname{Arom}_{(n)} (\operatorname{Arom}_{(n)} (A$	Ionition_of actions = { l. ∃a (location a = SOME l) } well_formed_action a = Gase a of	dependency undered_before actions threads location-kind aspanende before additional-synchronized with data-dependency control-dependency of modification-order release-aspanence corrito-a-dependency-to-a-b}	(b_chank_set is a > h_s beneficies > h_s
$ \begin{array}{l} \left(\operatorname{arcinulatif}_{(AVOM_{i}^{-1}) (AVOM_{i}^{-1}) (AOM_{i}^{-1}) = \operatorname{ad}_{j} \right) \wedge \\ \left(\operatorname{arcinulatif}_{(AVOM_{i}^{-1}) (AVOM_{i}^{-1}) = \operatorname{ad}_{j} \right) \wedge \\ \left(\operatorname{diventulatif}_{(AVOM_{i}^{-1}) (AVOM_{i}^{-1}) = \operatorname{ad}_{j} \right) \wedge \\ \end{array} \right) $	locations_of actions = (I. 3a (location z = Source I)) vvdLermed_actions z = case z = 0 Mrout_actions =	dependency_unlered_ledere actions thands location-kind appanende before additional-spectrosited with data-dependency costrol-dependency of modification-order release sequence carries-a-dependency-to a b) simple Jappene_Molece	$ \begin{array}{l} \left[b_{ad} tomi_{ad} extins \ A \ b_{ad} eq_{ad} t \ A \ b_{ad} t \ A \ b_{ad} eq_{ad} t \ A \ b_{ad} t \ A \ b_{ad} eq_{ad} t \ A \ b_{ad} t \ A \ b_{ad$
$ \begin{array}{l} (\operatorname{arises} \operatorname{dist} \left(\operatorname{Arous}_{i=1}, $	locations_of actions = (I. lis (location z = Souri I)) vvll_strunct_action z = case z = 0 (Mo_zent_action, Mo_action_	dependency_unitered_leftere actions timulas location kind aspanced before additional-spectromoder with data dependency control dependency of modification order relaxes aspance corries a dependency-to a b}	<pre>'(b_chank_set as A b_perform x A b_perf</pre>
$ \begin{array}{l} \left(\operatorname{articularial}_{(A \times \operatorname{COM}_{(A \times COM_{(A \times COM_{(A \times \operatorname{COM}_{(A \times \operatorname{COM}_{(A \times \operatorname{COM}_{(A \times \operatorname{COM}_{(A \times COM_{(A \times COM}_{(A \times COM_{(A \times $	Ionations_of actions = (I: 3a (Ionation a = 5000 f)) velLermod_actions =	bypendency_unitered_leftere actions threads, location kind auguenced before additional gendenoined with data dependency costeol dependency of modification-order release auguence corries a dependency-to a b) imple Auguent_Motor =	<pre>(h_chamin_setime a his h_set_mest as A h_</pre>
$ \begin{array}{l} \left(\operatorname{arises}_{i=0} d_i \left(\operatorname{Arous}_{i=1} A (\operatorname{Arous}_{i=1} d_i \right) \land \\ \left(\operatorname{arises}_{i=0} d_i \left(\operatorname{Arous}_{i=1} (\operatorname{Arous}_{i=1} d_i \right) \land \\ \left(\operatorname{arises}_{i=0} d_i d_i \left(\operatorname{Arous}_{i=1} d_i d_i \right) \land \\ \left(\operatorname{arises}_{i=0} d_i d_i \right) \left(\operatorname{Arous}_{i=1} d_i d_i - d_i \right) \land \\ \left(\operatorname{arises}_{i=0} d_i d_i \left(\operatorname{Arous}_{i=1} d_i d_i - d_i \right) \land \\ \left(\operatorname{bread,dat}_{i=0} (\operatorname{Arous}_{i=1} d_i d_i - d_i d_i \right) \land \\ \left(\operatorname{bread,dat}_{i=0} (\operatorname{Arous}_{i=1} d_i d_i - d_i d_i \right) \land \\ \left(\operatorname{bread,dat}_{i=0} (\operatorname{Arous}_{i=1} d_i d_i - d_i d_i \right) \land \\ \left(\operatorname{bread,dat}_{i=0} (\operatorname{Arous}_{i=1} d_i d_i d_i d_i d_i d_i d_i d_i d_i d_i$	<pre>locations_of actions = (I. Ba (location z = South f)) vull_formul_action z= coses z of Arous_locationmem.ordmem.ord ∈ (Mo_stata.cost_Mo_stata.co</pre>	begreakers; underscherduleder zeiten tansals bezein skind zugenende befere additional operkonnierd with data dependency control dependency of modification order release augunera: control - dependency - to a b}	<pre> (p_duning_state a / h_genergy A = A = A = A = A = A = A = A = A = A</pre>
	<pre>locations_of actions = (I. 3a (location z = South f)) vvll_transl_action z = case z = 0 vvll_transl_action z = f(A_1, A_2, A_3, A_4, A_4, A_4, A_4, A_4, A_4, A_4, A_4</pre>	bipendancy_unitered_lefter actions threads location shift assumed before additional gendenoised with data dependency costeol dependency of modification-order release assumer corries a dependency-to a b) simple Asspects. before = simple Asspects. before = simple Asspects. before = before additional gendenoised with data dependency costeol dependency of modification-order release asspector corries a dependency-to a b) simple Asspects. before = simple Assert additional gendenoised with data dependency costeol dependency of modification-order release asspector corries a dependency-to a b) simple Asspects. before = simple Assert additional gendenoised with data dependency costeol dependency of modification-order release asspector corries a dependency-to a b) simple Asspects. before = simple Assert additional gendency and the simple Assert additional gendency additional	<pre>in (priority state as his (prior h) (happender as h) have the as h) (happender as h) is (priority - final hostion his degeneration as h) is (priority - final hostion his degeneration as h) is (priority - final hostion his degeneration has h) is (priority - final hostion his degeneration has defended on the hostion his degeneration of the assessment hast degeneratory certed degeneratory / is (priority - final hostion his degeneration has defended on the hostion his degeneration certed degeneratory / is (priority - final hostion his degeneration has defended on the hostion his degeneration certed degeneratory / is (priority - final hostion his degeneration has defended on the hostion his degeneration certed degeneratory / is (priority - final hostion his degeneration has defended on the hostion his degeneration has degeneration certed degeneratory / is (priority - final hostion his degeneration has defended on the hostion his degeneration has degeneratory certed degeneratory / is (priority - final hostion his degeneration has degenerated where additional generation defenders / is (priority - final hostion his degeneration has degenerated where additional generation where a set addition of the hostion his degeneration (priority - final hostion his degeneration has degenerated where additional generation where a set addition has degenerated where additional generation has degenerated and the addition of the addition of the hostion his degeneration has degenerated where additional generation has degenerated and degree model has addition degree model has addition degree to hostion has degree to hostion has degre</pre>
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$ \begin{cases} \text{(sciencial of (Arounc-1on 2 \text{ of }) = ad) \land \\ (\text{(sciencial of (Arounc-1on 2 \text{ of }) = ad) \land \\ (\text{(sciencial of (Arounc-1on 2 \text{ or }$	<pre>locations_of actions = (I. 3a (location a = Source f)) vvdLormeLaction a = case a d f(A) = actions = memory = memory = f(A) = memory = f(A) = memory = f(A) = memory = f(A) = memory = memory = f(A) = memory = memory = f(A) = memory = memory = f(A) = memory = f(A) = memory = memor</pre>	by producty, and model, we actions themasks location kind sequences before additional synchronoised with data dependency control dependency of modification order relaxes sequence corries a dependency-to a b) function of the sequence of	<pre> function sets as A for a for A sequence as A for a for A sequence as A for A sequence A for A sequence as A for A sequence A for A sequence</pre>
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$ \begin{bmatrix} (\operatorname{restinuid_of}(\operatorname{ArounAnon 2d}, \dots, -) = dd \rangle \land \\ (\operatorname{restinuid_of}(\operatorname{ArounAnon 2d}, \dots, -) = dd) \land \\ (\operatorname{restinuid_odd}(\operatorname{ArounAnon 2d}, \dots, -) = dd) \land \\ (\operatorname{restinuid_odd}(\operatorname{ArounAnon 2d}, \dots, -) = dd) \land \\ (\operatorname{restinuid_odd}(\operatorname{ArounAnon 2d}, \dots, -) = dd) \land \\ (\operatorname{restinuid_odd}(\operatorname{Aroun}, -) = dd) \land \\ (\operatorname{restinuid_odd}(\operatorname{Aroun}, -) = dd) \land \\ (\operatorname{restinuid_odd}(\operatorname{Aroun}, -) = dd) \land \\ (\operatorname{restinuid_oddd}(\operatorname{Aroun}, -) = dd) \land \\ (\operatorname{restinuid_oddd}(Aroun$	$\label{eq:second} \begin{split} & \operatorname{location} = (I, \exists a \ (\operatorname{location} a = \operatorname{Soute} \ h)) \\ & \operatorname{vell.formulation} a = \\ & \operatorname{vell.formulation} (A_{A_{A_{A_{A_{A_{A_{A_{A_{A_{A_{A_{A_{A$	<pre>depredenty_setMent_lefer action threak bacatos hid appended befor additional question and adjusted only of modification order mixes equance carries a dipendency to a b) fingle depresent deform =</pre>	<pre>planet state state</pre>
$ \begin{bmatrix} (\operatorname{scienced}_{(1,0)} (\operatorname{Around}_{(2,0)} \operatorname{Around}_{(2,0)} A$	$ \begin{aligned} & \text{locations}_{\text{eff}} (I, \exists a \in [0, a \in [0, \exists a \in [0, a a \in [0, a a \in [0, a a [$	<pre>deproducty_undersd_before actions that apparend before additional quenches with data dependency on action dependency on a bit principle deproduction of a measure data apparend before additional quenches and with data dependency of modification order relates apparent data appa</pre>	<pre>planet state state</pre>
$ \begin{bmatrix} (\operatorname{sciencial_of}_{(\operatorname{const}, -1)} & = d_0 \land \\ (\operatorname{sciencial_of}_$	$\begin{aligned} & \left[\operatorname{continue_of} a \operatorname{strine} = (I, \exists a (\operatorname{contine} a = \operatorname{Soute} h) \right] \\ & \left[\operatorname{continue_of} a \operatorname{strine} a = \operatorname{contine} a = \operatorname{contine} h \right] \\ & \left[\operatorname{continue_of} a = \operatorname{contine} a = \operatorname{contine} h \right] \\ & \left[\operatorname{continue_of} a = \operatorname{contine} a = \operatorname{contine} h \right] \\ & \left[\operatorname{continue} a = \operatorname{contine} a = \operatorname{contine} h \right] \\ & \left[\operatorname{contine} a = \operatorname{contine} a = \operatorname{contine} h \right] \\ & \left[\operatorname{contine} a = \operatorname{contine} a = \operatorname{contine} a = \operatorname{contine} h \right] \\ & \left[\operatorname{contine} a = \operatorname{contine} a =$	<pre>deproducty_undersd_before actions that apparend before additional quedrouslaw with data dependency of modification order minans apparent carries a dependency to a b) function (</pre>	<pre>planet state state</pre>

Is C++11 hopelessly complicated?

Programmers cannot be given this model.

However, with a formal definition, **we can do proofs**! For instance:

- Can we compile to x86?	Operation load(non-seq_ load(seq_cst) store(non-seq_ store(seq_cst) fence(non-seq_	_cst) mov lock xchg
- Can we compile to Power?	C++0x Operation Non-atomic Load Load Relaxed Load Consume Load Acquire Load Seq Cst	POWER Implementation 1d 1d 1d (and preserve dependency) 1d; cmp; bc; isync sync; 1d; cmp; bc; isync
andour 0. Jonuoru 17	Non-atomic Store Store Relaxed Store Release Store Seq Cst	st st lwsync; st sync; st

Is C++11 hopelessly complicated?

Simplifications:

Full model: visible sequences of side effects are unneeded (HOL4) Derivative models:

- without consume, happens-before is transitive

The current state of the standard

Fixed:

- in some cases, happens-before was cyclic
- coherence
- seq_cst atomics were more broken

Not fixed:

- out of thin air reads (and self satisfying conditionals)
- **seq_cst** atomics do not guarantee SC





4. Hunting compiler concurrency bugs



```
int a = 1;
int b = 0;
```

Thread 1

```
int s;
for (s=0; s!=4; s++) {
    if (a==1)
      return NULL;
    for (b=0; b>=26; ++b)
      ;
}
```

Thread 2

```
int a = 1;
int b = 0;
```

Thread 1

Thread 2

```
int s;
for (s=0; s!=4; s++) {
    if (a==1)
        return NULL;
    for (b=0; b>=26; ++b)
      ;
}
```

```
b = 42;
printf("%d\n", b);
```

```
int a = 1;
int b = 0;
```

Thread 1

```
int s;
for (s=0; s!=4; s++) {
    if (a==1)
       return NULL;
    for (b=0; b>=26; ++b)
      ;
}
```

Thread 2

b = 42;
printf("%d\n", b);

```
int a = 1;
int b = 0;
```

Thread 1

Thread 2

int s;
for (s=0; s!=4; s++) {
 if (a==1)
 return NULL;
 for (b=0; b>=26; ++b)
 ;
}

b = 42;
printf("%d\n", b);

```
int a = 1;
int b = 0;
```

Thread 1

Thread 2

```
int s;
for (s=0; s!=4; s++) {
    if (a==1)
        return NULL;
    for (b=0; b>=26; ++b)
      ;
}
```

```
b = 42;
printf("%d\n", b);
```

Thread 1

```
int s;
for (s=0; s!=4; s++) {
    if (a==1)
        return NULL;
    for (b=0; b>=26; ++b)
      ;
}
```

Thread 2

```
int a = 1;
int b = 0;
```

Thread 1

Thread 2

Thread 1 returns without modifying b.

```
int a = 1;
int b = 0;
```

Thread 1

Thread 2

```
int s;
for (s=0; s!=4; s++) {
    if (a==1)
       return NULL;
    for (b=0; b>=26; ++b)
       ;
}
```

```
b = 42;
printf("%d\n", b);
```

Thread 1 returns without modifying b.

```
int a = 1;
int b = 0;
```

Thread 1

```
int s;
for (s=0; s!=4; s++) {
    if (a==1)
      return NULL;
    for (b=0; b>=26; ++b)
      ;
}
```

Thread 2

```
int a = 1;
int b = 0;
```

Thread 1

```
int s;
for (s=0; s!=4; s++) {
    if (a==1)
        return NULL;
    for (b=0; b>=26; ++b)
      ;
}
```

Thread 2

```
int a = 1;
int b = 0;
```

Thread 1

```
int s;
for (s=0; s!=4; s++) {
    if (a==1)
       return NULL;
    for (b=0; b>=26; ++b)
      ;
}
```

Thread 2



...sometimes we get 0 on the screen

```
int s;
for (s=0; s!=4; s++) {
    if (a==1)
       return NULL;
    for (b=0; b>=26; ++b)
      ;
}
```

```
movl a(%rip), %edx # load a into edx
movl b(%rip), %eax # load b into eax
testl %edx, %edx # if a!=0
jne .L2 # jump to .L2
movl $0, b(%rip)
ret
.L2:
movl %eax, b(%rip) # store eax into b
xorl %eax, %eax # store 0 into eax
ret # return
```

The outer loop can be (and is) optimised away

movl	a(%rip), %edx	# load a into edx
movl	b(%rip), %eax	<pre># load b into eax</pre>
testl	%edx, %edx	# if a!=0
jne	.L2	# jump to .L2
movl	\$0, b(%rip)	
ret		
.L2:		
movl	<pre>%eax, b(%rip)</pre>	<pre># store eax into b</pre>
xorl	%eax, %eax	<pre># store 0 into eax</pre>
ret		# return

movl	a(%rip), %edx	<pre># load a into edx</pre>
movl	b(%rip), %eax	<pre># load b into eax</pre>
testl	%edx, %edx	# if a!=0
jne	.L2	# jump to .L2
movl	\$0, b(%rip)	
ret		
.L2:		
movl	<pre>%eax, b(%rip)</pre>	<pre># store eax into b</pre>
xorl	%eax, %eax	<pre># store 0 into eax</pre>
ret		# return

movl	a(%rip), %edx	<pre># load a into edx</pre>
movl	b(%rip), %eax	<pre># load b into eax</pre>
testl	%edx, %edx	# if a!=0
jne	.L2	# jump to .L2
movl	\$0, b(%rip)	
ret		
.L2:		
movl	<pre>%eax, b(%rip)</pre>	<pre># store eax into b</pre>
xorl	%eax, %eax	<pre># store 0 into eax</pre>
ret		# return

movl	a(%rip), %edx	<pre># load a into edx</pre>
movl	b(%rip), %eax	<pre># load b into eax</pre>
testl	%edx, %edx	# if a!=0
jne	.L2	# jump to .L2
movl	\$0, b(%rip)	
ret		
.L2:		
movl	%eax, b(%rip)	<pre># store eax into b</pre>
xorl	%eax, %eax	<pre># store 0 into eax</pre>
ret		# return

movl	a(%rip), %edx	<pre># load a into edx</pre>
movl	b(%rip), %eax	<pre># load b into eax</pre>
testl	%edx, %edx	# if a!=0
jne	.L2	# jump to .L2
movl	\$0, b(%rip)	
ret		
.L2:		
movl	<pre>%eax, b(%rip)</pre>	<pre># store eax into b</pre>
-		
xorl	%eax, %eax	<pre># store 0 into eax</pre>

```
movl a(%rip), %edx  # load a into edx
movl b(%rip), %eax  # load b into eax
testl %edx, %edx  # if a!=0
jne .L2  # jump to .L2
movl $0, b(%rip)
ret
.L2:
movl %eax, b(%rip)  # store eax into b
xorl %eax, %eax  # store 0 into eax
ret  # return
```

The compiled code saves and restores **b**

Correct in a sequential setting.

What about concurrency?

movl	a(%rip), %edx	<pre># load a into edx</pre>
movl	b(%rip), %eax	<pre># load b into eax</pre>
testl	%edx, %edx	# if a!=0
jne	.L2	# jump to .L2
movl	\$0, b(%rip)	
ret		
.L2:		
movl	<pre>%eax, b(%rip)</pre>	<pre># store eax into b</pre>
xorl	%eax, %eax	<pre># store 0 into eax</pre>
ret		# return

```
int a = 1;
int b = 0;
```

Thread 1

```
int s;
for (s=0; s!=4; s++) {
    if (a==1)
      return NULL;
    for (b=0; b>=26; ++b)
      ;
}
```

Thread 2

```
int a = 1;
int b = 0;
```

Thread 1

Thread 2

Since Thread 1 does not update b, program is data-race free (DRF)

```
int a = 1;
int b = 0;
```

Thread 1

Thread 2

Since Thread 1 does not update b, program is *data-race free (DRF)* DRF programs must only exhibit sequentially consistent behaviours *C11/C++11 standard*

```
int a = 1;
int b = 0;
```

Thread 1

Thread 2

Since Thread 1 does not update b, program is *data-race free (DRF)* DRF programs must only exhibit sequentially consistent behaviours *C11/C++11 standard*

This program MUST only print 42.

```
int a = 1;
int b = 0;
```

Thread 1

```
int s;
for (s=0; s!=4; s++) {
    if (a==1)
      return NULL;
    for (b=0; b>=26; ++b)
      ;
}
```

Thread 2

```
int a = 1;
int b = 0;
```

Thread 1

movl a(%rip),%edx
movl b(%rip),%eax
testl %edx, %edx
jne .L2
movl \$0, b(%rip)
ret
.L2:
movl %eax, b(%rip)
xorl %eax, %eax
ret

Thread 2

b = 42;
printf("%d\n", b);

```
int a = 1;
int b = 0;
```

Thread 1

movl	a(%rip),%edx
movl	b(%rip),%eax
testl	%edx, %edx
jne	.L2
movl	\$0, b(%rip)
ret	
L2:	
movl	%eax, b(%rip)
xorl	%eax, %eax
ret	

Thread 2

b = 42;
printf("%d\n", b);

- Read a (1) into edx

```
int a = 1;
int b = 0;
```

Thread 1

movl	a(%rip),%edx
movl	b(%rip),%eax
testl	%edx, %edx
jne	.L2
movl	\$0, b(%rip)
ret	
L2:	
movl	%eax, b(%rip)
xorl	%eax, %eax
ret	

Thread 2

b = 42;
printf("%d\n", b);

- Read a (1) into edx - Read b (0) into eax

```
int a = 1;
int b = 0;
```

Thread 1

movl a(%rip),%edx
movl b(%rip),%eax
testl %edx, %edx
jne .L2
movl \$0, b(%rip)
ret
.L2:
movl %eax, b(%rip)
xorl %eax, %eax
ret

Thread 2

b = 42;
printf("%d\n", b);

- Read a (1) into edx
- Read b (0) into eax
- Store 42 into b

```
int a = 1;
int b = 0;
```

Thread 1

a(%rip),%edx movl movl b(%rip),%eax testl %edx, %edx jne .L2 \$0, b(%rip) movl ret .L2: %eax, b(%rip) movl %eax, %eax xorl ret

Thread 2

- Read a (1) into edx
- Read b (0) into eax
- Store 42 into b
- Store eax (0) into b

```
int a = 1;
int b = 0;
```

Thread 1

a(%rip),%edx movl movl b(%rip),%eax testl %edx, %edx jne .L2 \$0, b(%rip) movl ret .L2: movl %eax, b(%rip) xorl %eax, %eax ret

Thread 2

- Read a (1) into edx
- Read b (0) into eax
- Store 42 into b
- Store eax (0) into b
- Print b... 0 is printed

The compiled code saves and restores b Correct in a sequential setting Introduces unexpected behaviours in some concurrent context

ret

.L2:

movl %eax, b(%rip) xorl %eax, %eax ret

- Read a (1) into edx
- Read b (0) into eax
- Store 42 into b
- Store eax (0) into b
- Print b... 0 is printed

The compiled code saves and restores b

Correct in a sequential setting

Introduces unexpected behaviours in some concurrent context

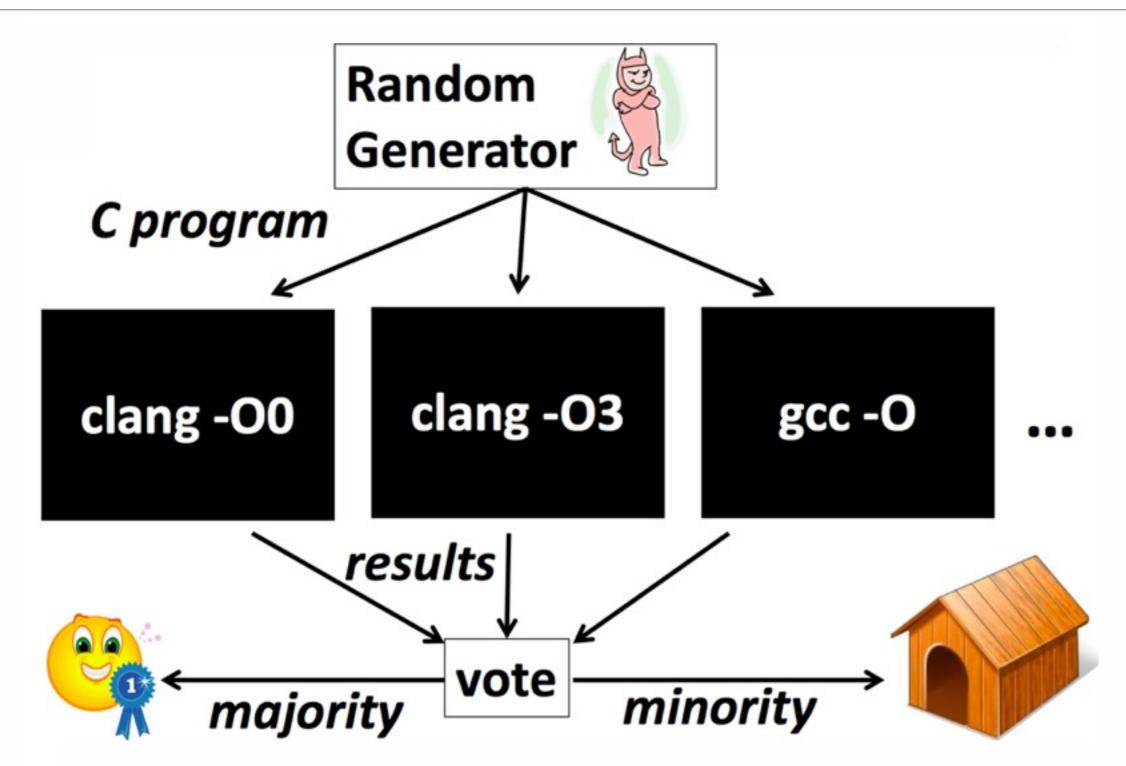
This is a concurrency compiler bug

movl %eax, b(%rıp) xorl %eax, %eax ret

- Read b (0) into eax
- Store 42 into b
- Store eax (0) into b
- Print b... 0 is printed

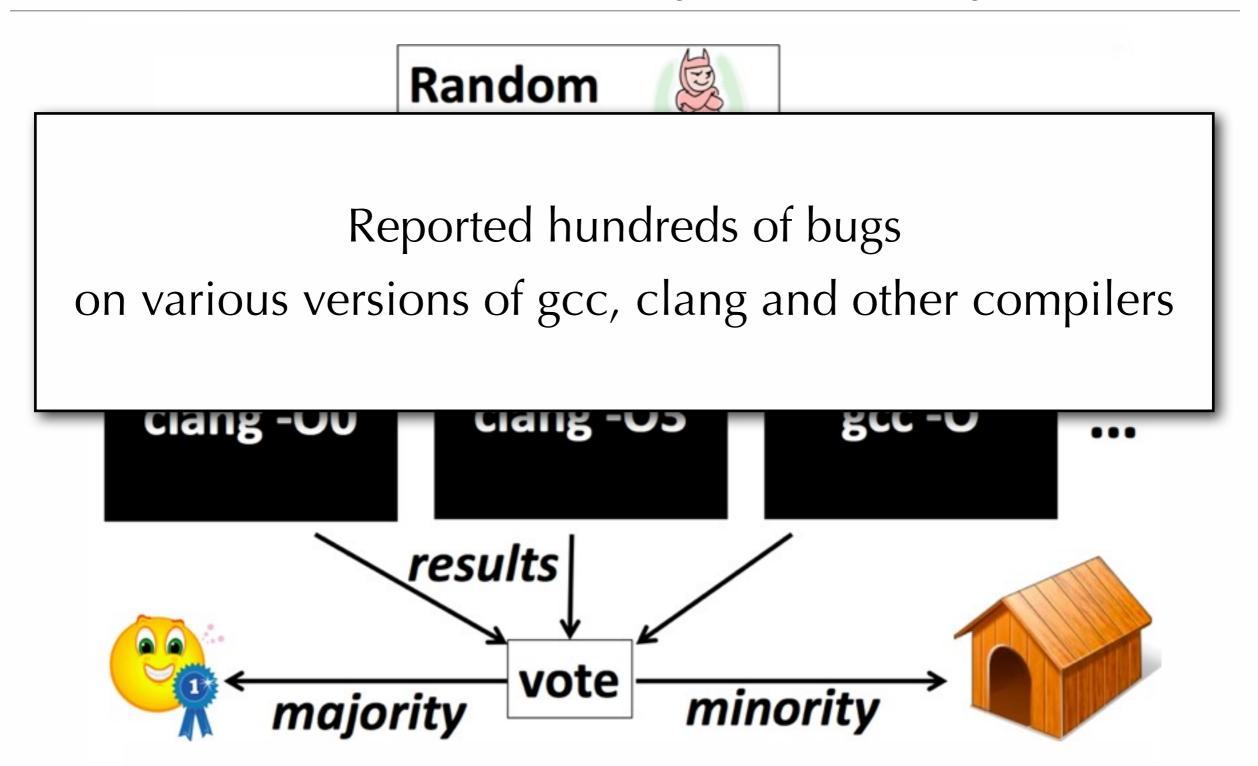
Compiler testing: state of the art

Yang, Chen, Eide, Regehr - PLDI 2011



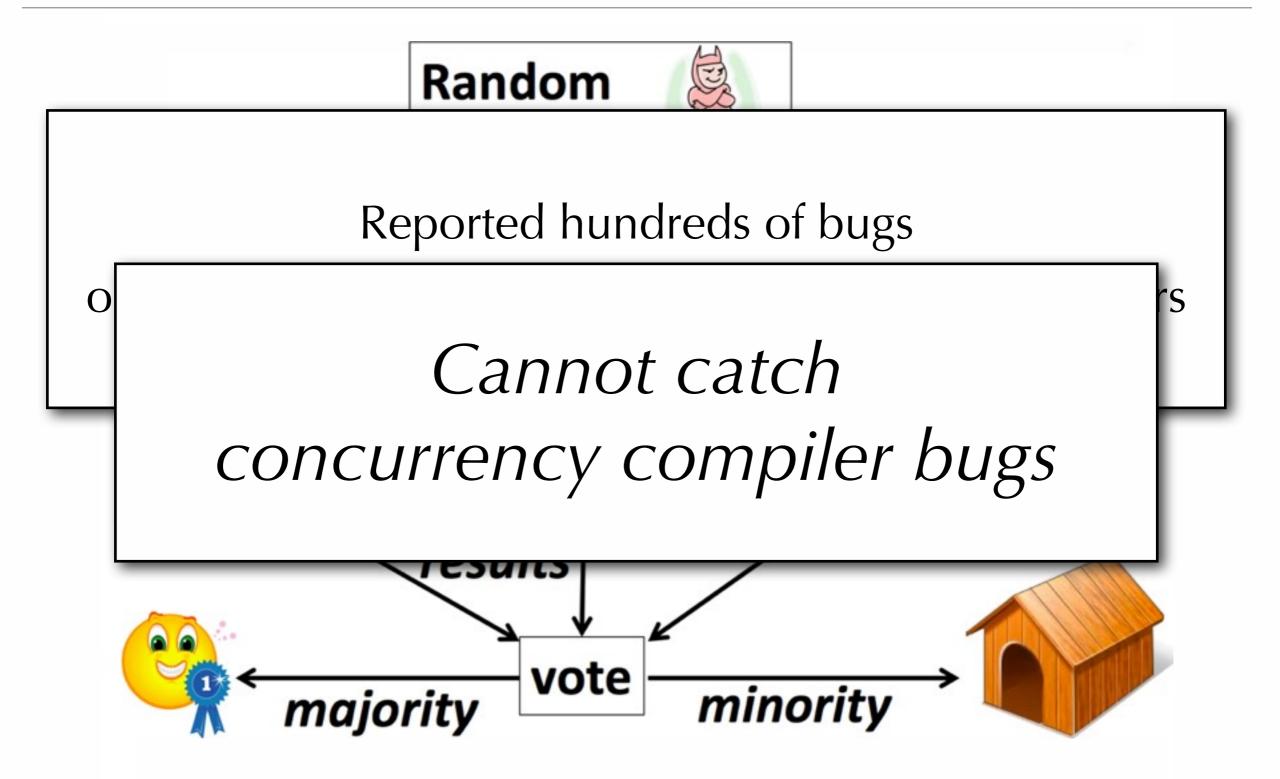
Compiler testing: state of the art

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Compiler testing: state of the art

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Hunting concurrency compiler bugs?

How to deal with non-determinism?

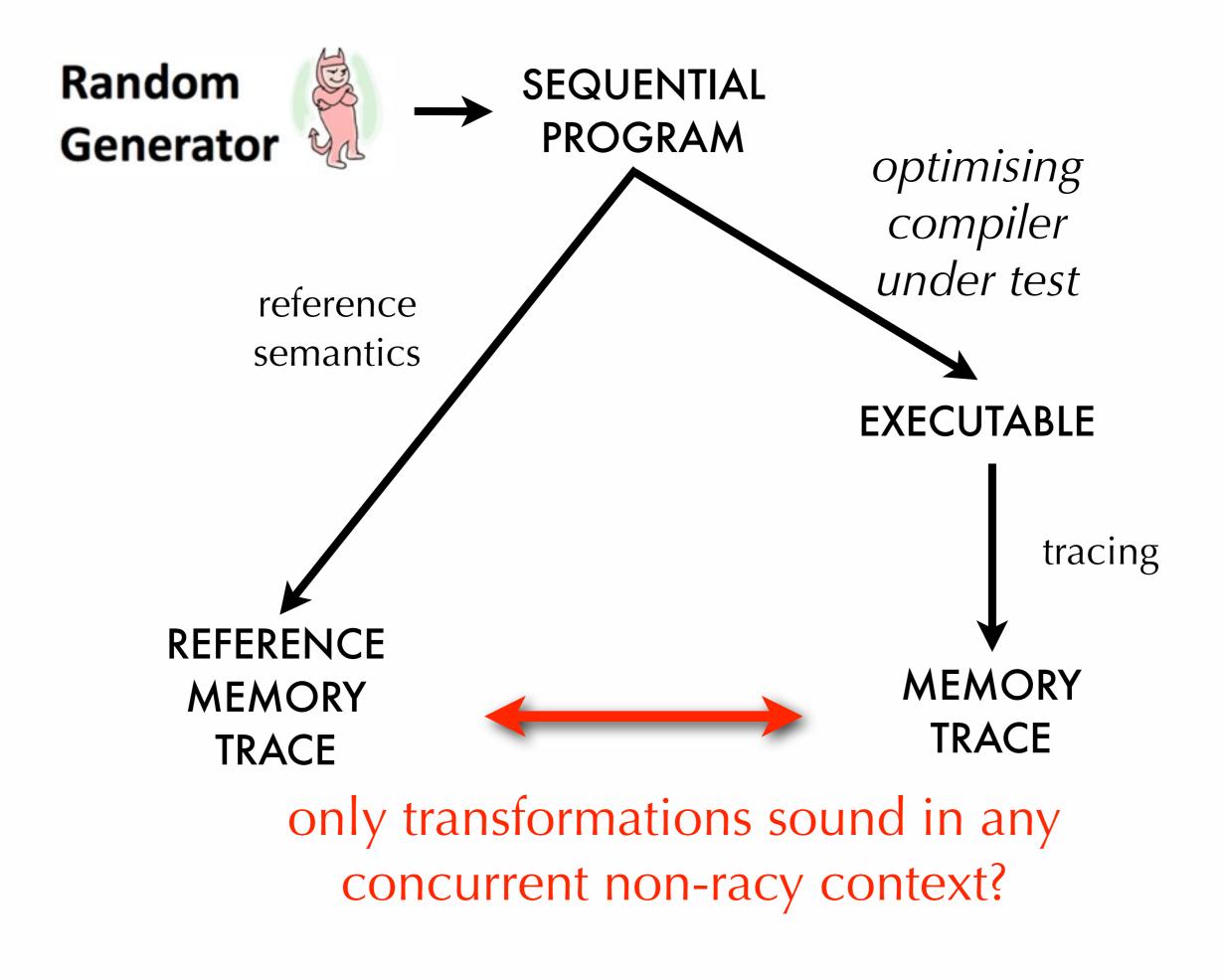
How to generate non-racy interesting programs?

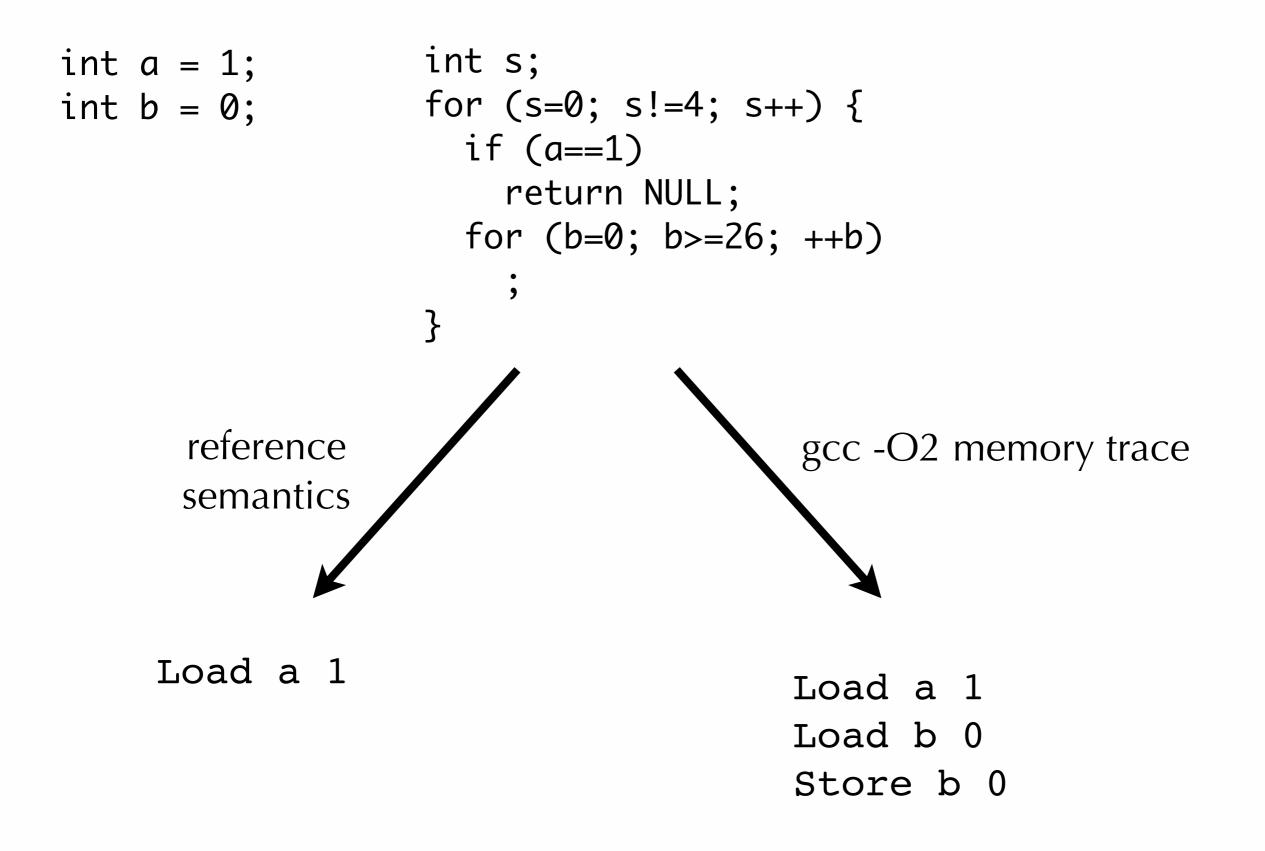
How to capture all the behaviours of concurrent programs?

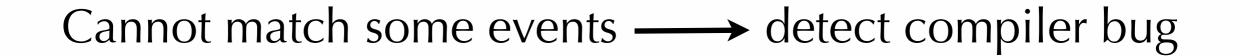
A compiler can optimise away behaviours: *how to test for correctness? limit case*: two compilers generate correct code with disjoint final states C/C++ compilers support separate compilation Functions can be called in arbitrary non-racy concurrent contexts C/C++ compilers can only apply transformations sound with respect to an arbitrary non-racy concurrent context

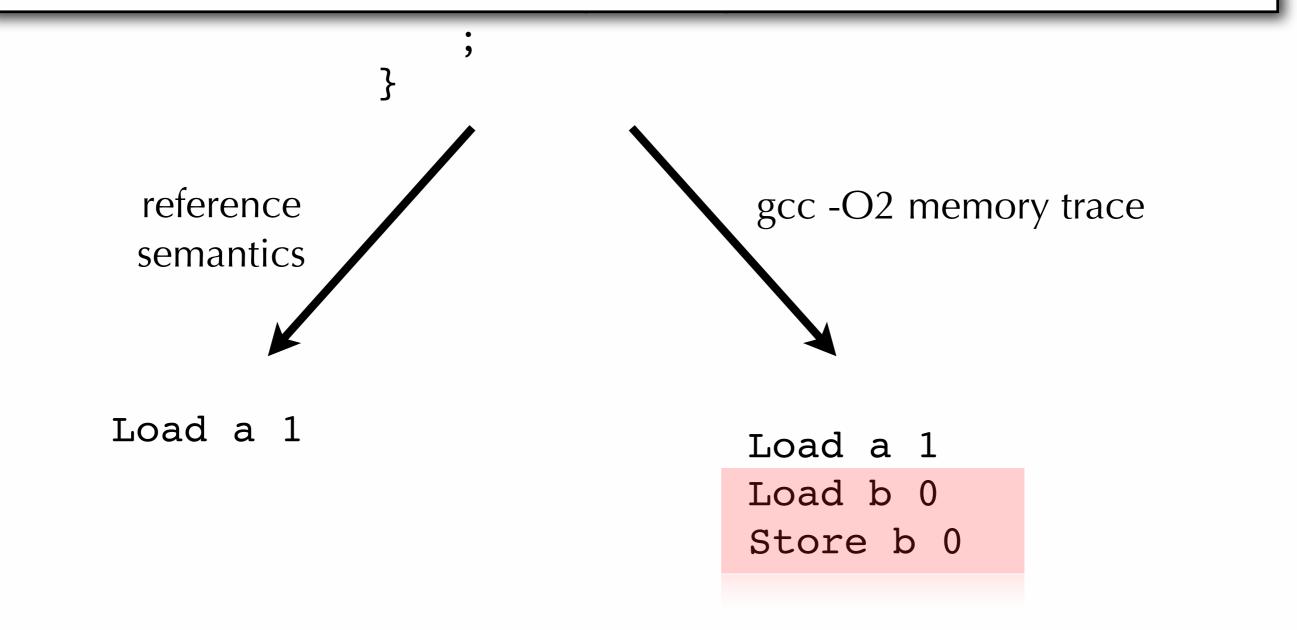
Hunt concurrency compiler bugs

search for transformations of sequential code not sound in an arbitrary non-racy context









Contributions

Sound optimisations in the C11/C++11 memory model extending Sevcik's work on an idealised DRF model - PLDI 11

A tool to hunt concurrency bugs in C and C++ compilers

Interaction with GCC developers

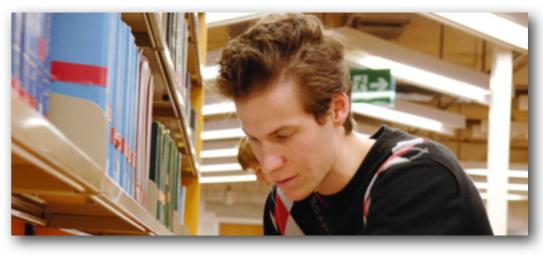
Sound Optimisations in the C11/C++11 Memory Model

Example: loop invariant code motion

Compiler Writer



Semanticist



Example: loop invariant code motion

Compiler Writer



Sophisticated program analyses Fancy algorithms Source code or IR

Operations on AST

Semanticist



Example: loop invariant code motion

Compiler Writer



Sophisticated program analyses Fancy algorithms Source code or IR

Operations on AST

Semanticist



Compiler Writer



Sophisticated program analyses Fancy algorithms Source code or IR

Operations on AST

Semanticist



```
tmp = y+1 ;
for (int i=0; i<2; i++) {
   z = i;
   x[i] +=tmp;
}</pre>
```

Compiler Writer



Sophisticated program analyses Fancy algorithms Source code or IR

Operations on AST

Semanticist



Elimination of run-time events Reordering of run-time events Introduction of run-time events *Operations on sets of events*

```
tmp = y+1 ;
for (int i=0; i<2; i++) {
   z = i;
   x[i] +=tmp;
}</pre>
```

Compiler Writer



Sophisticated program analyses Fancy algorithms Source code or IR

Operations on AST

Semanticist



Elimination of run-time events Reordering of run-time events Introduction of run-time events *Operations on sets of events*

```
Store z 0
Load y 42
Store x[0] 43
Store z 1
Load y 42
Store x[1] 43
```

Compiler Writer



Sophisticated program analyses Fancy algorithms Source code or IR

Operations on AST

Semanticist



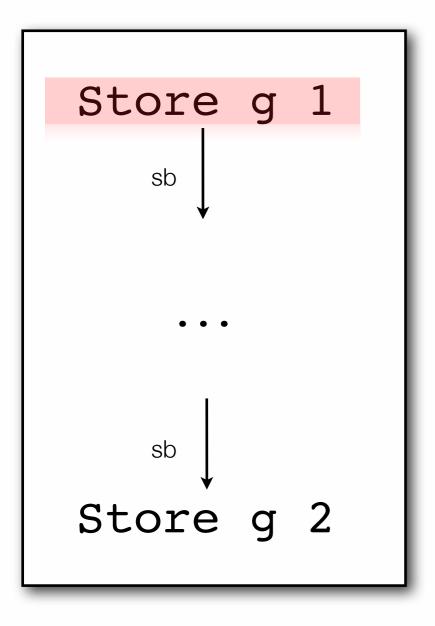
Elimination of run-time events Reordering of run-time events Introduction of run-time events Operations on sets of events

> Load y 42 Store z 0

Store x[0] 43 Store z 1

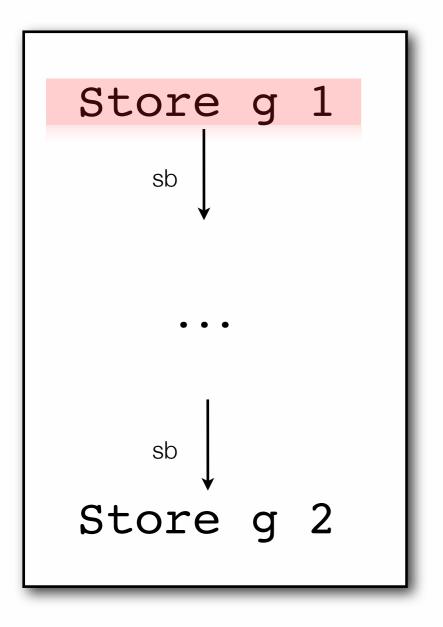
Store x[1] 43

Elimination of overwritten writes



Under which conditions is it correct to eliminate the first store?

Elimination of overwritten writes



Under which conditions is it correct to eliminate the first store?

What is the semantics of C11/C++11 concurrent code?

The C11/C++11 memory model

C11/C++11 are based on the DRF approach:

- racy code is undefined
- race-free code must exhibit only sequentially consistent behaviours
- main synchronisation mechanism: lock/unlock

Escape mechanism for experts, low-level atomics:

- races allowed
- attributes on accesses specify their semantics:

MO_SEQ_CST MO_RELEASE/MO_ACQUIRE MO_RELAXED

$$g = 0$$
; atomic $f = 0$;

Thread 1

Thread 2

g = 42; f.store(1,MO_RELEASE);

$$g = 0;$$
 atomic $f = 0;$

Thread 1

Thread 2

g = 42; f.store(1,MO_RELEASE);

$$g = 0;$$
 atomic $f = 0;$

Thread 1

Thread 2

g = 42; f.store(1,MO_RELEASE);

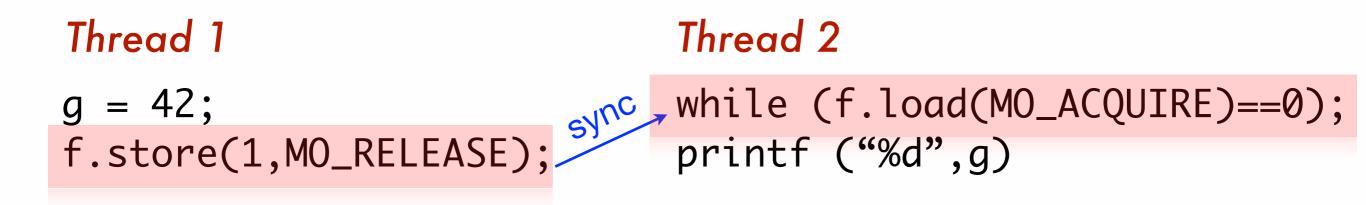
$$g = 0;$$
 atomic $f = 0;$

Thread 1

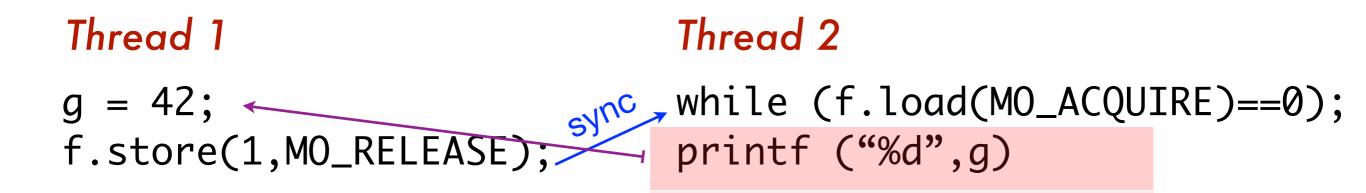
Thread 2

g = 42; f.store(1,MO_RELEASE);

$$g = 0;$$
 atomic $f = 0;$



$$g = 0$$
; atomic $f = 0$;



The release/acquire synchronisation guarantees that:

- the program is DRF
- 42 is printed at the end of the execution

Remark: unlock \approx release, lock \approx acquire.

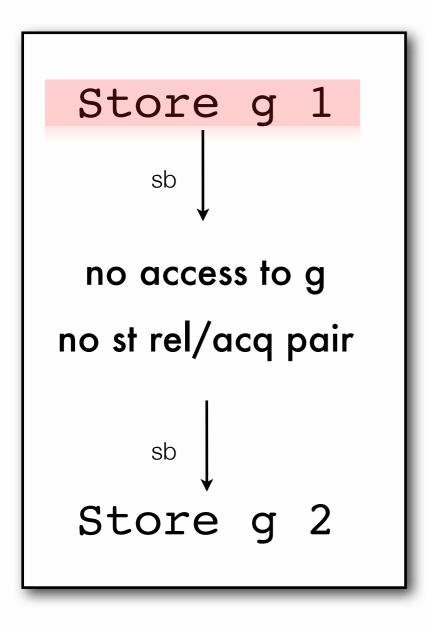
Same-thread release/acquire pairs

A same-thread release-acquire pair is a pair of a release action followed by an acquire action in program order.

An action is a *release* if it is a possible source of a synchronisation *unlock mutex, release or seq_cst atomic write*

An action is an *acquire* if it is a possible target of a synchronisation lock mutex, acquire or seq_cst atomic read

Elimination of overwritten writes



It is safe to eliminate the first store if there are:

 no intervening accesses to **g** no intervening same-thread release-acquire pairs

Shared memory

$$g = 0;$$
 atomic f1 = f2 = 0;

Thread 1

```
g = 1;
f1.store(1,RELEASE);
while(f2.load(ACQUIRE)==0);
g = 2;
```

Shared memory

$$g = 0$$
; atomic f1 = f2 = 0;

Thread 1 candidate overwritten write
g = 1;
f1.store(1,RELEASE);
while(f2.load(ACQUIRE)==0);
g = 2;

Shared memory

$$g = 0;$$
 atomic f1 = f2 = 0;

Thread 1 candidate overwritten write
g = 1;
f1.store(1,RELEASE); same-thread release-acquire pair
while(f2.load(ACQUIRE)==0);
g = 2;

Shared memory

$$g = 0$$
; atomic f1 = f2 = 0;

Thread 1

```
g = 1;
f1.store(1,RELEASE);
while(f2.load(ACQUIRE)==0);
g = 2;
```

Thread 2

while(f1.load(ACQUIRE)==0);
printf("%d", g);
f2.store(1,RELEASE);

Shared memory

$$g = 0;$$
 atomic f1 = f2 = 0;

Thread 1

Thread 2

g = 1; f1.store(1,RELEASE); while(f1.load(ACQUIRE)==0); while(f2.load(ACQUIRE)==0); g = 2; while(f1.load(ACQUIRE)==0); f2.store(1,RELEASE);

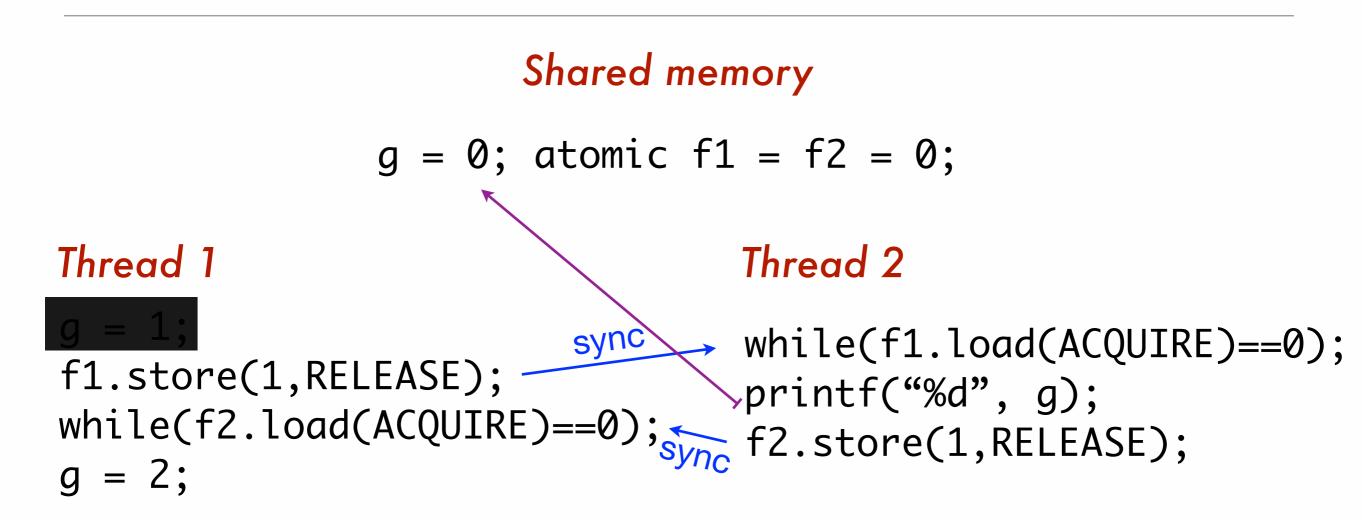
Thread 2 is non-racy

Shared memory

$$g = 0;$$
 atomic f1 = f2 = 0;

Thread 1 Thread 2 g = 1; f1.store(1,RELEASE); while(f1.load(ACQUIRE)==0); while(f2.load(ACQUIRE)==0); f2.store(1,RELEASE); g = 2;

Thread 2 is non-racy The program should only print **1**



Thread 2 is non-racy The program should only print **1**

If we perform overwritten write elimination it prints 0

Shared memory

$$g = 0$$
; atomic f1 = f2 = 0;

Thread 1

Thread 2

while(f1.load(ACQUIRE)==0);
printf("%d", g);
f2.store(1,RELEASE);

Shared memory

$$g = 0;$$
 atomic f1 = f2 = 0;

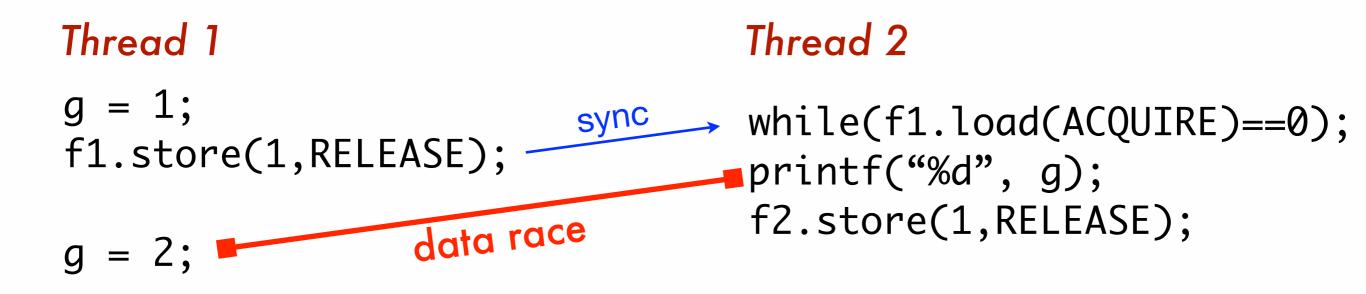
Thread 1

Thread 2

while(f1.load(ACQUIRE)==0);
printf("%d", g);
f2.store(1,RELEASE);

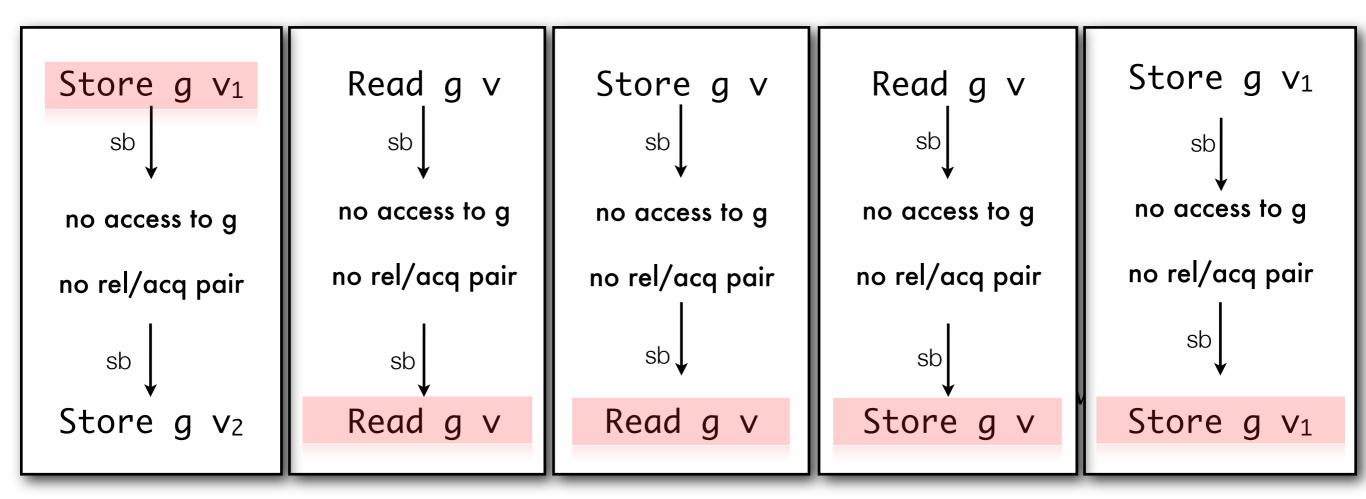
Shared memory

$$g = 0;$$
 atomic f1 = f2 = 0;



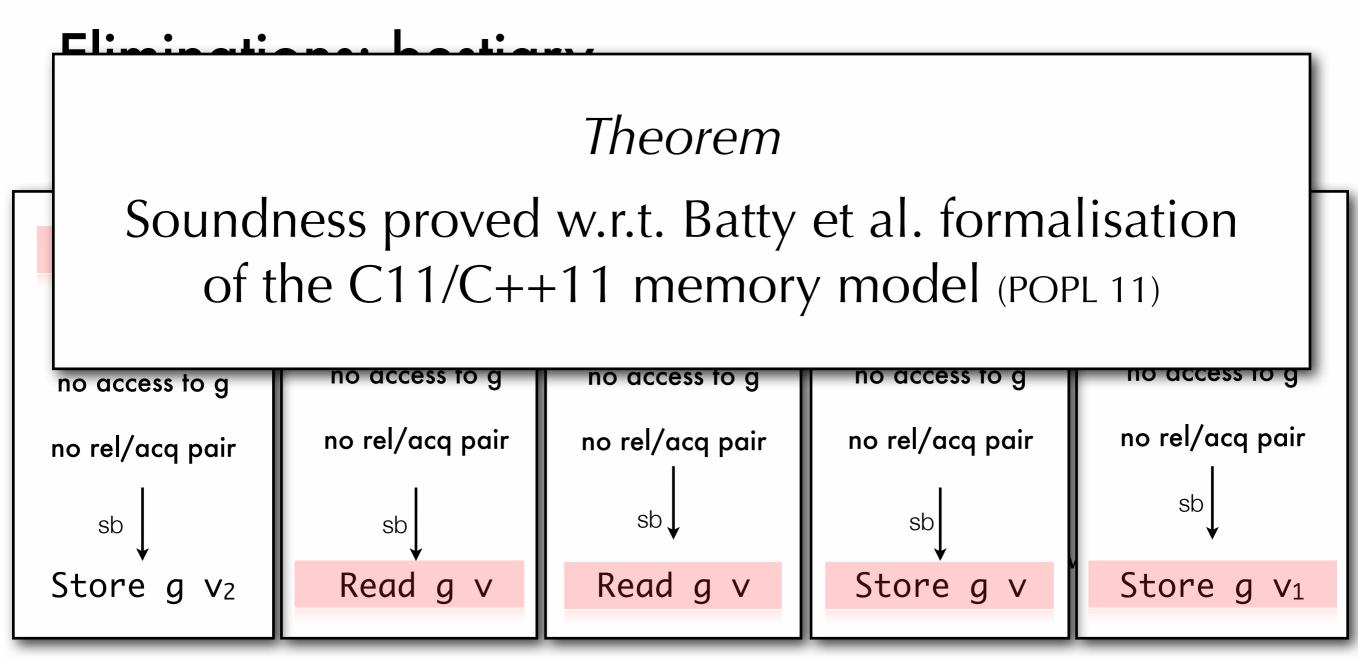
If only a release (or acquire) is present, then all discriminating contexts *are racy*. It is sound to optimise the overwritten write.

Eliminations: bestiary



Overwritten-Write Read-after-Read Read-after-Write Write-after-Read Write-after-Write

Reads which are not used (via data or control dependencies) to decide a write or synchronisation event are also eliminable (*irrelevant reads*).



Overwritten-Write Read-after-Read Read-after-Write Write-after-Read Write-after-Write

Reads which are not used (via data or control dependencies) to decide a write or synchronisation event are also eliminable (*irrelevant reads*).

Reorderings and introductions

Correctness criterion for reordering events:

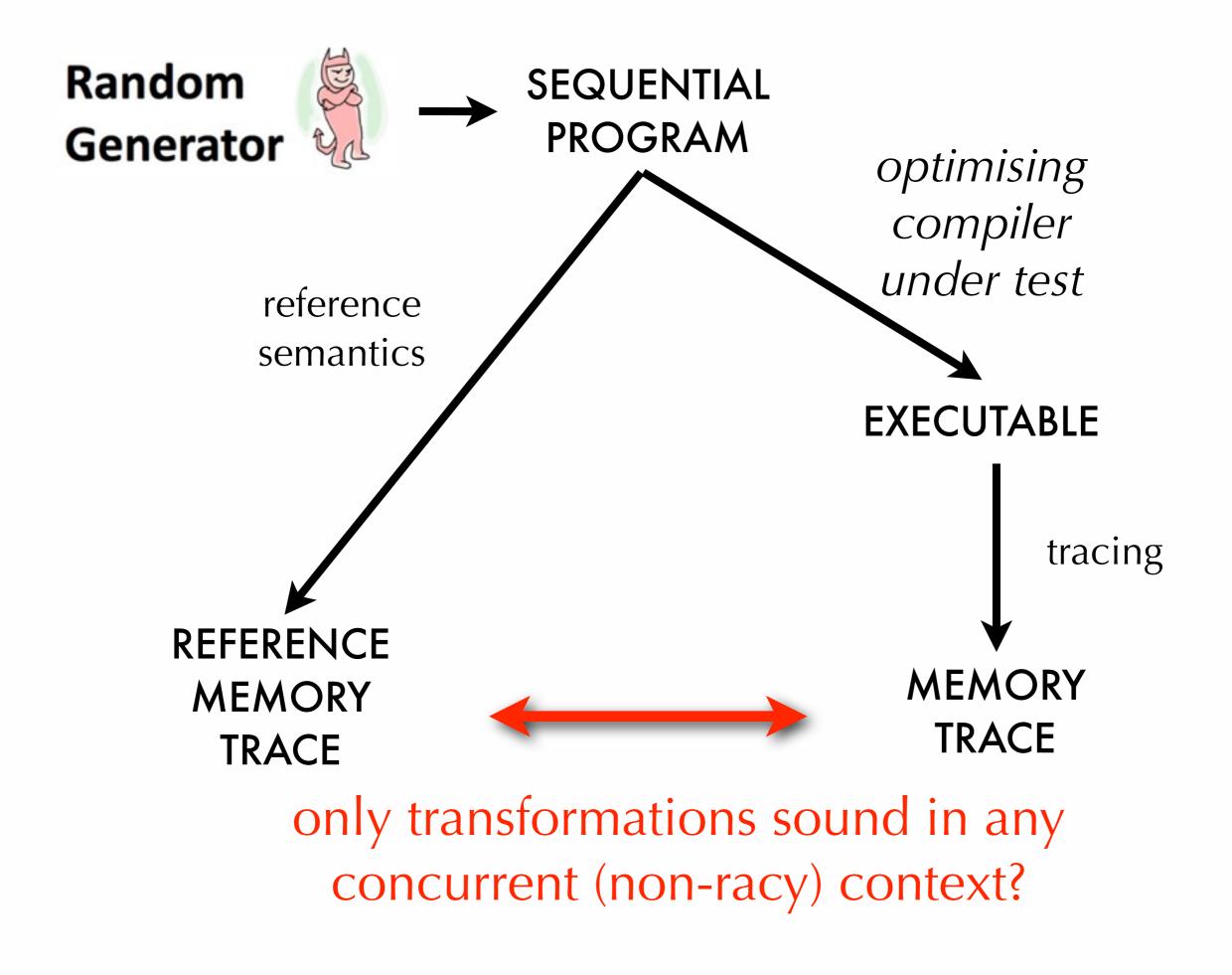
- different addresses
- no synchronisations in-between

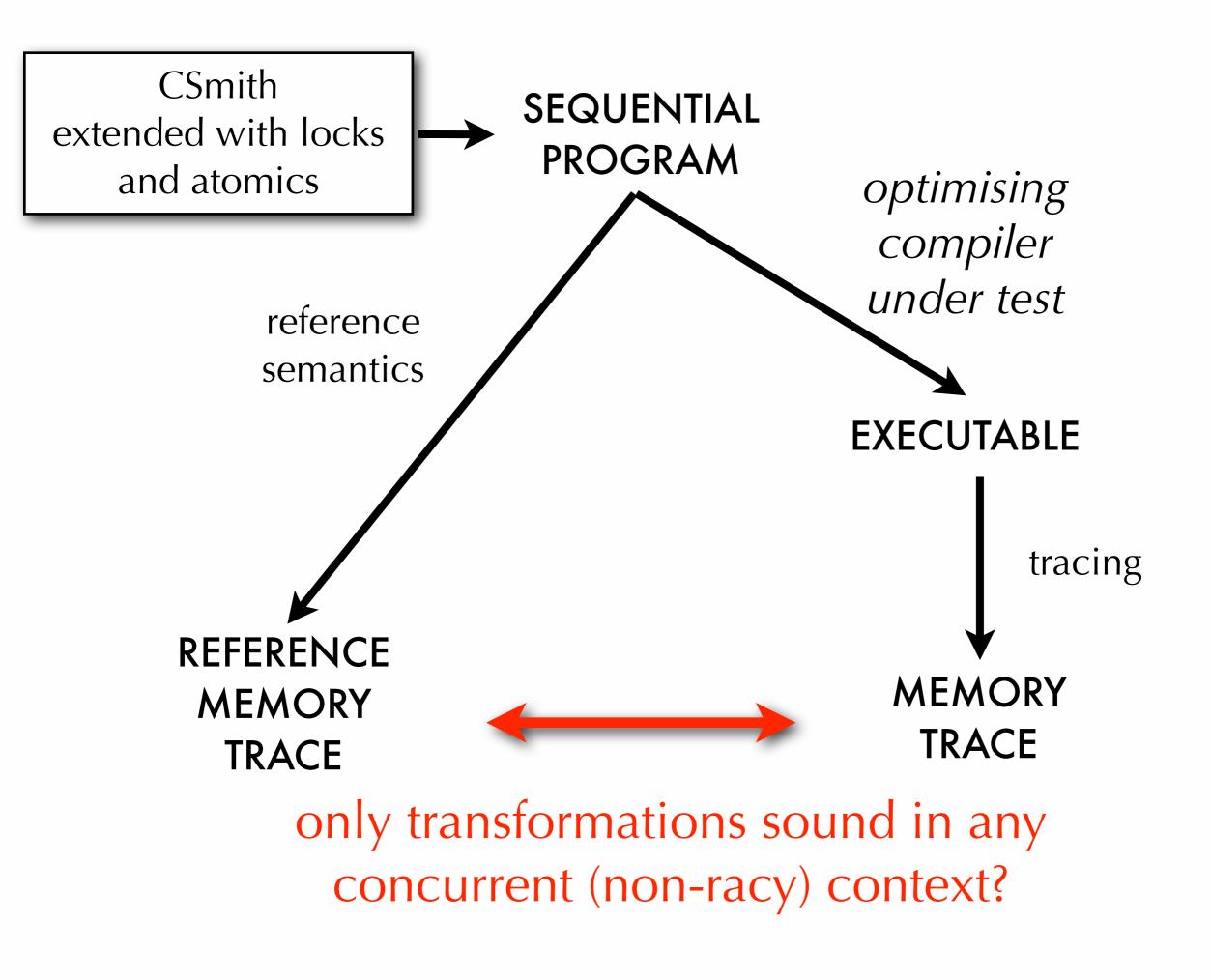
Roach-motel reordering (reordering across locks) not observed in practice

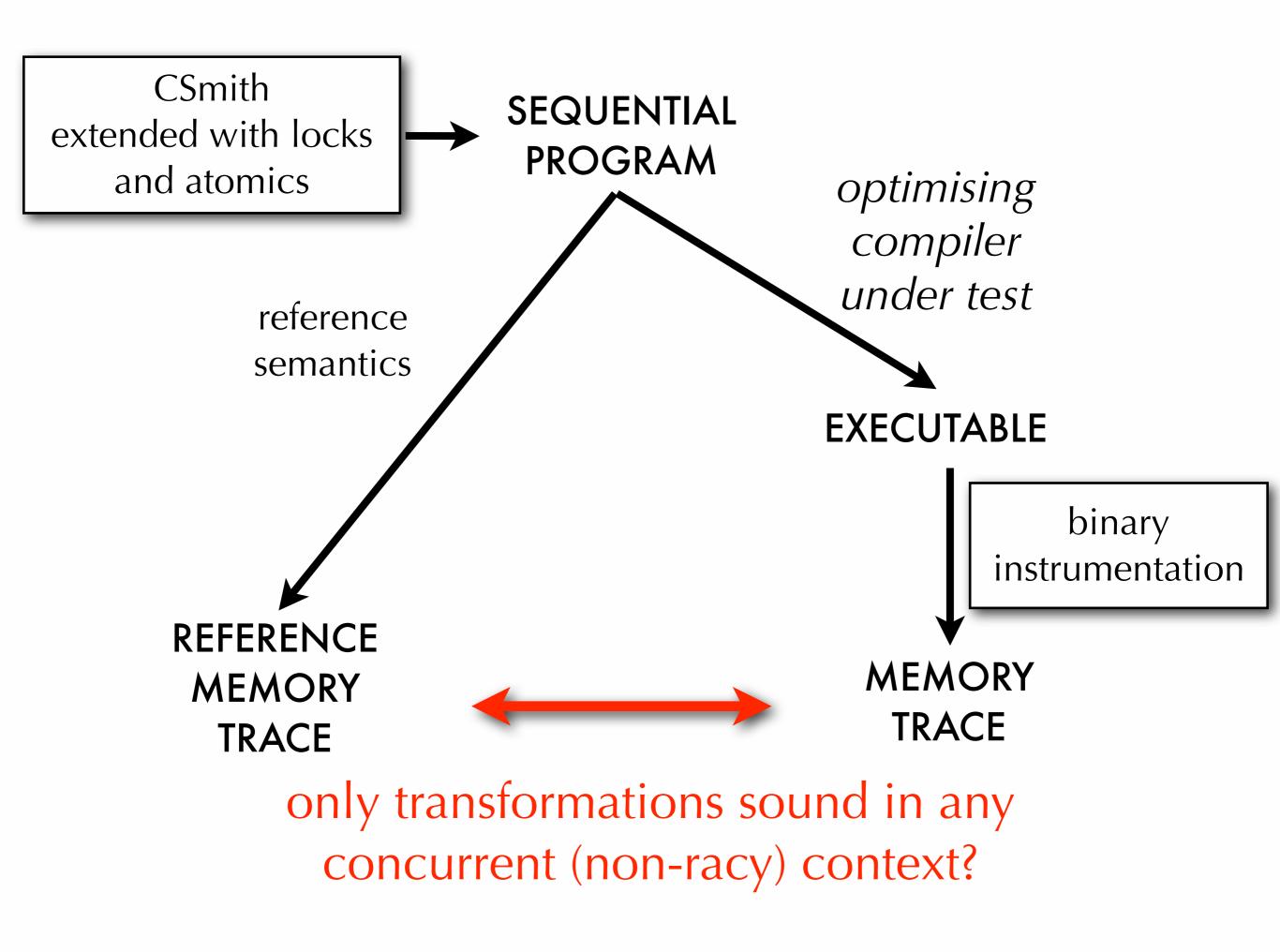
Read introductions observed in practice (gcc, clang).

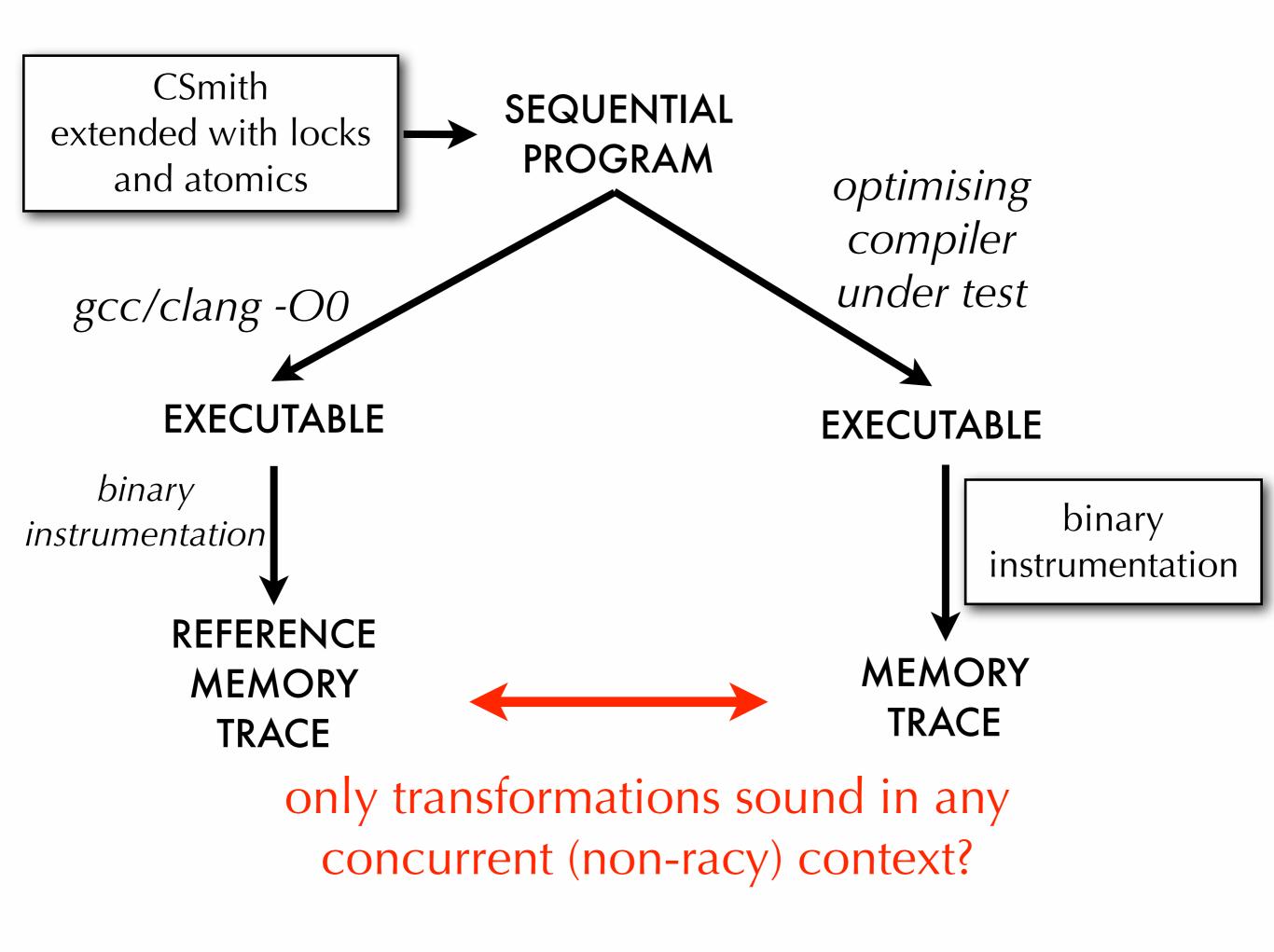
Introduction of eliminable reads proved correct. Introduction of irrelevant reads does not introduce new behaviours, but cannot be proved correct in a DRF model.

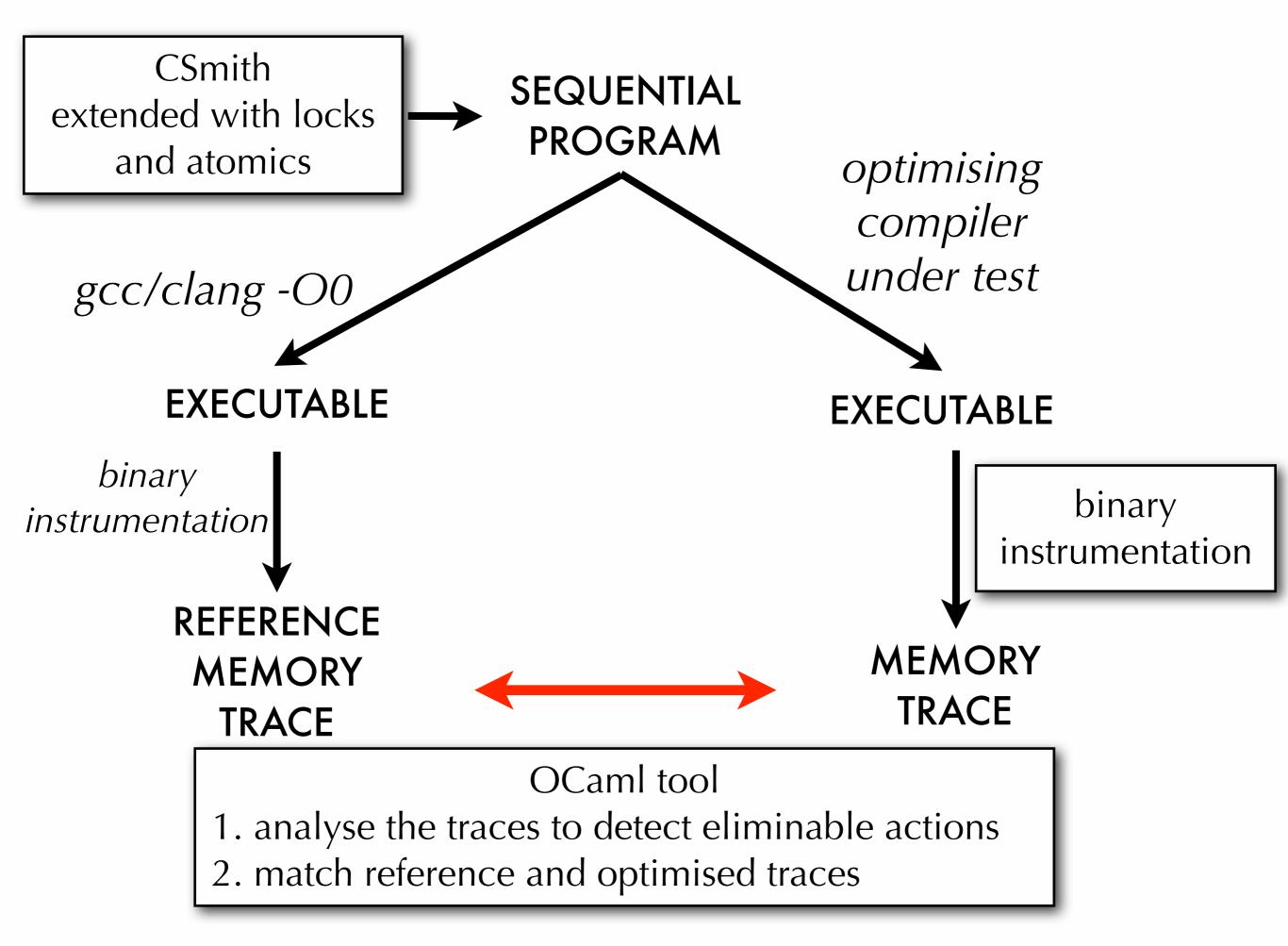
The CMMTEST Tool

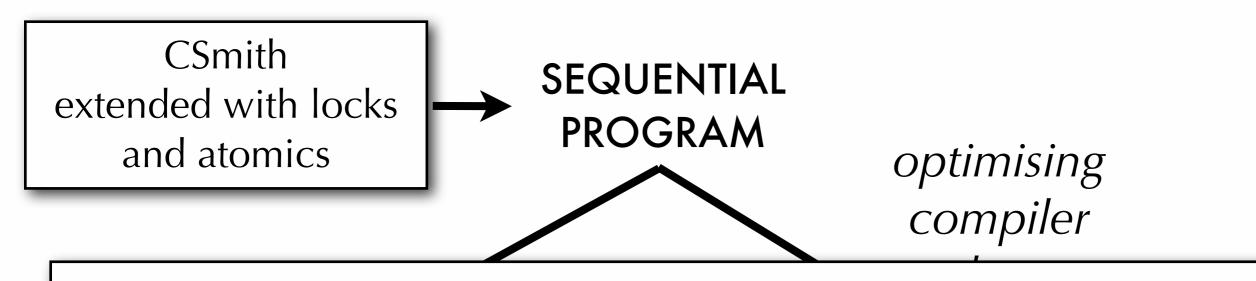












Subtleties:

- dependencies between eliminable events
- some optimisations (e.g. merging of accesses) cannot be expressed in the C11/C++11 formalisation
- the tool also ensures that the compilation of atomic accesses is preserved by the optimiser

OCaml tool

1. analyse the traces to detect eliminable actions

2. match reference and optimised traces

Ir

Interaction with GCC developers

1. Some GCC bugs

Some concurrency compiler bugs found in the latest version of GCC.

Store introductions performed by loop invariant motion or if-conversion optimisations.

All promptly fixed.

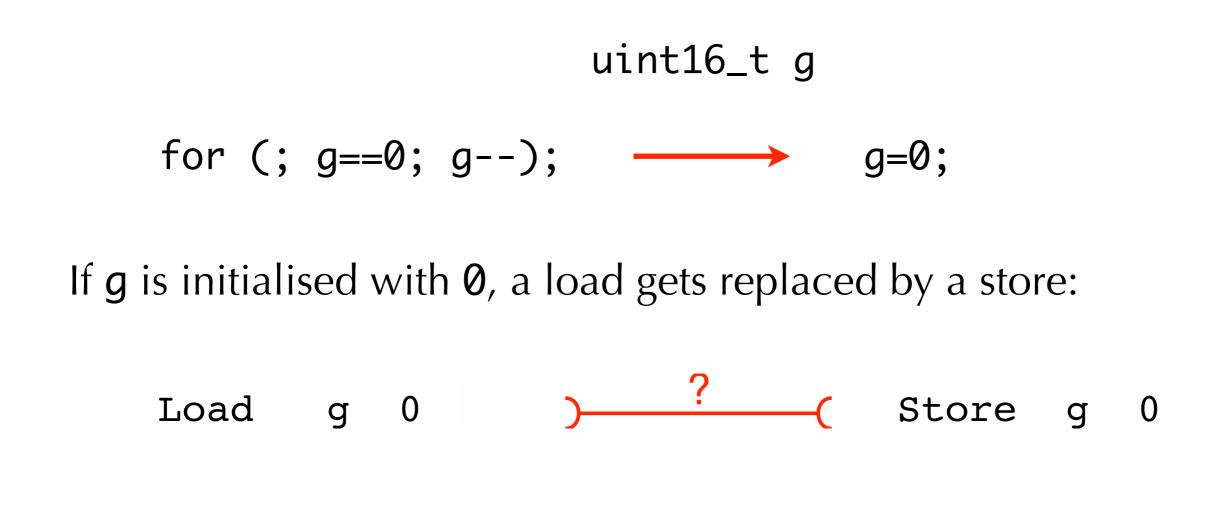
Remark: these bugs break the Posix thread model too.

2. Checking compiler invariants

GCC internal invariant: never reorder with an atomic access

Baked this invariant into the tool and found a counterexample... ...not a bug, but fixed anyway

3. Detecting unexpected behaviours



The introduced store cannot be observed by a non-racy context. Still, *arguable if a compiler should do this or not*.

Conclusion

Syllabus



In these lectures we have covered the hardware models of two modern computer architectures (x86 and Power/ARM - at least for a large subset of their instruction set).

We have seen how compiler optimisations can also break concurrent programs and the importance of defining the memory model of highlevel programming languages.

We have also introduced some proof methods to reason about concurrency.

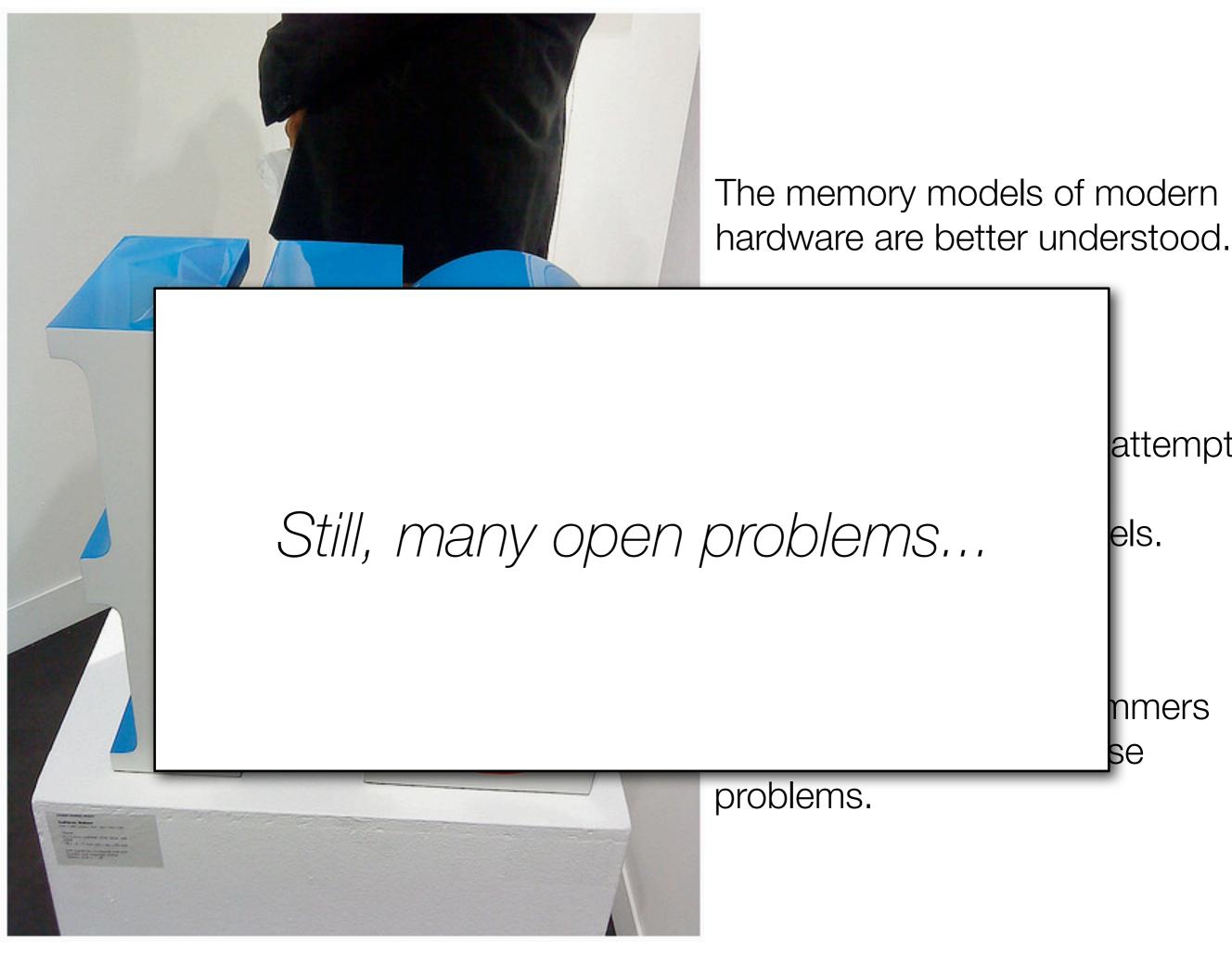
After these lectures, you might have the feeling that multicore programming is a mess and things can't just work.



The memory models of modern hardware are better understood.

Programming languages attempt to specify and implement reasonable memory models.

Researchers and programmers are now interested in these problems.

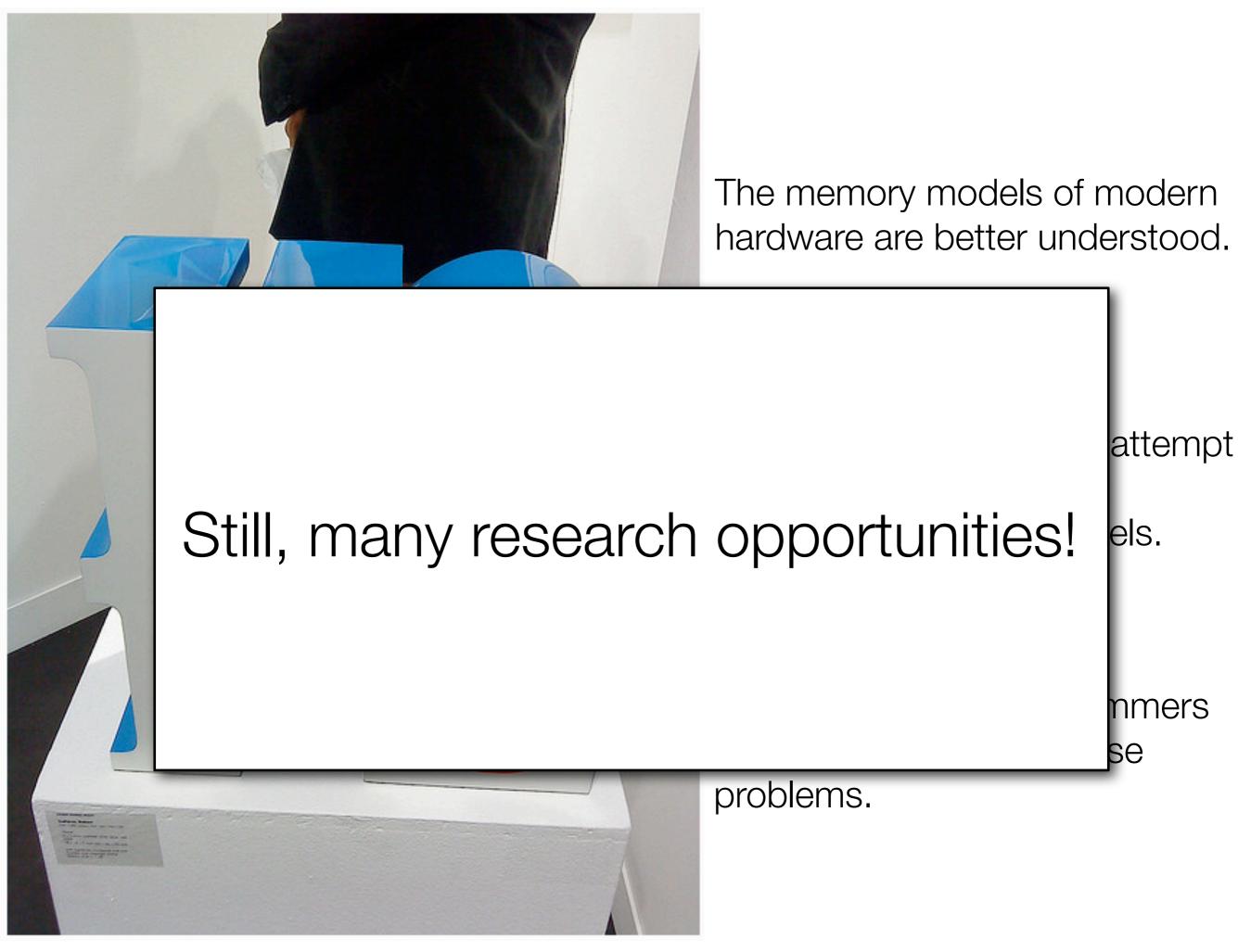


attempt

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All these lectures are based on work done with/by my colleagues. Thank you!





And thank you all for attending these lectures!

Please, fill the course evaluation form, that's important to make a better course next year.



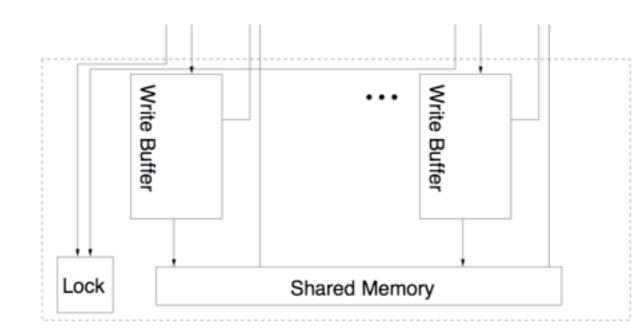


4. Sketch of an operational formalisation of x86-TSO

...starting with a formalisation of SC

Separate language and memory semantics

```
class ArrayWrapper
 2
     {
 3
         public:
 4
             ArrayWrapper (int n)
 5
                  : p vals( new int[ n ] )
 6
                  , size(n)
 7
             {}
 8
             // copy constructor
 9
             ArrayWrapper (const ArrayWrapper& other)
10
                  : p vals( new int[ other. size ] )
11
                  , _size( other._size )
12
13
                  for ( int i = 0; i < size; ++i )</pre>
14
                      p vals[ i ] = other. p vals[ i ];
16
17
18
             ~ArrayWrapper ()
19
20
                  delete [] p vals;
21
22
         private:
23
         int * p vals;
24
         int size;
25
     };
```

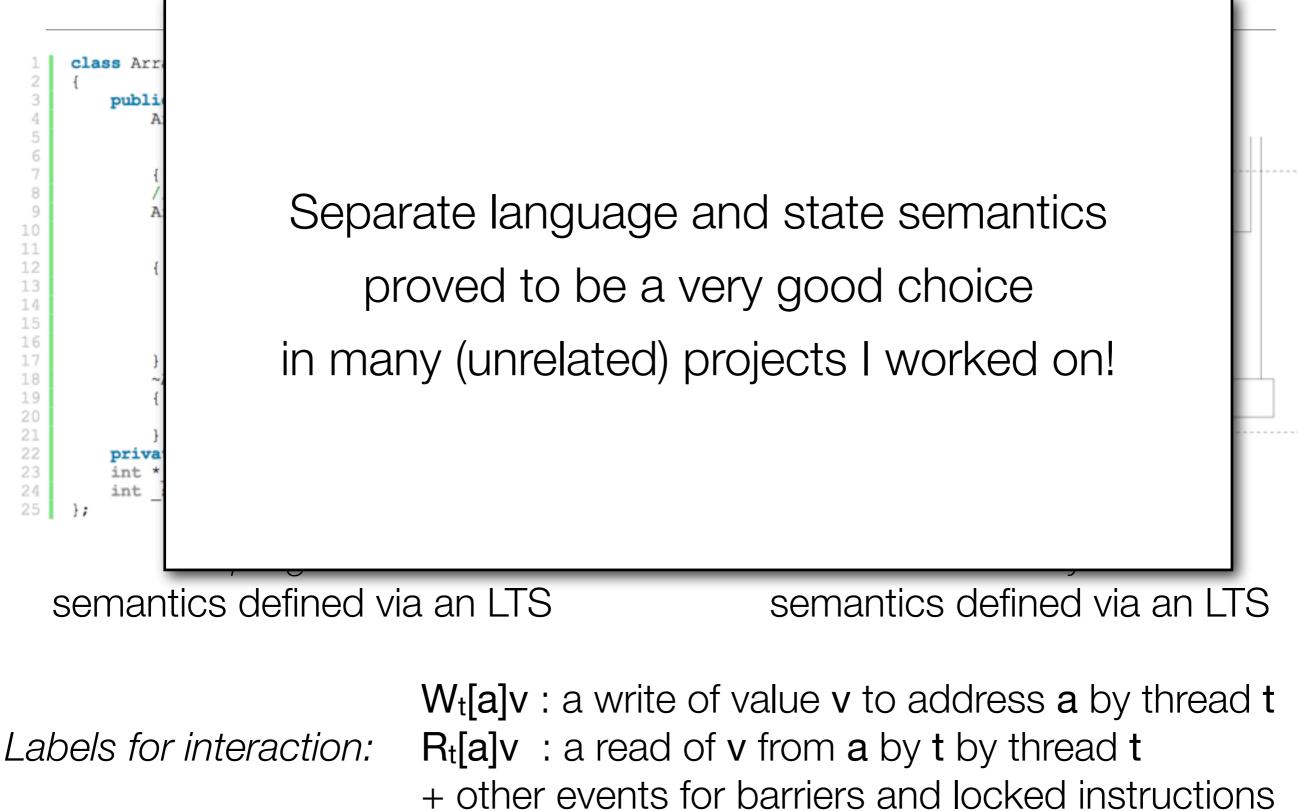


program memory semantics defined via an LTS semantics defined via an LTS

Labels for interaction:

Wt[a]v : a write of value v to address a by thread t
Rt[a]v : a read of v from a by t by thread t
+ other events for barriers and locked instructions

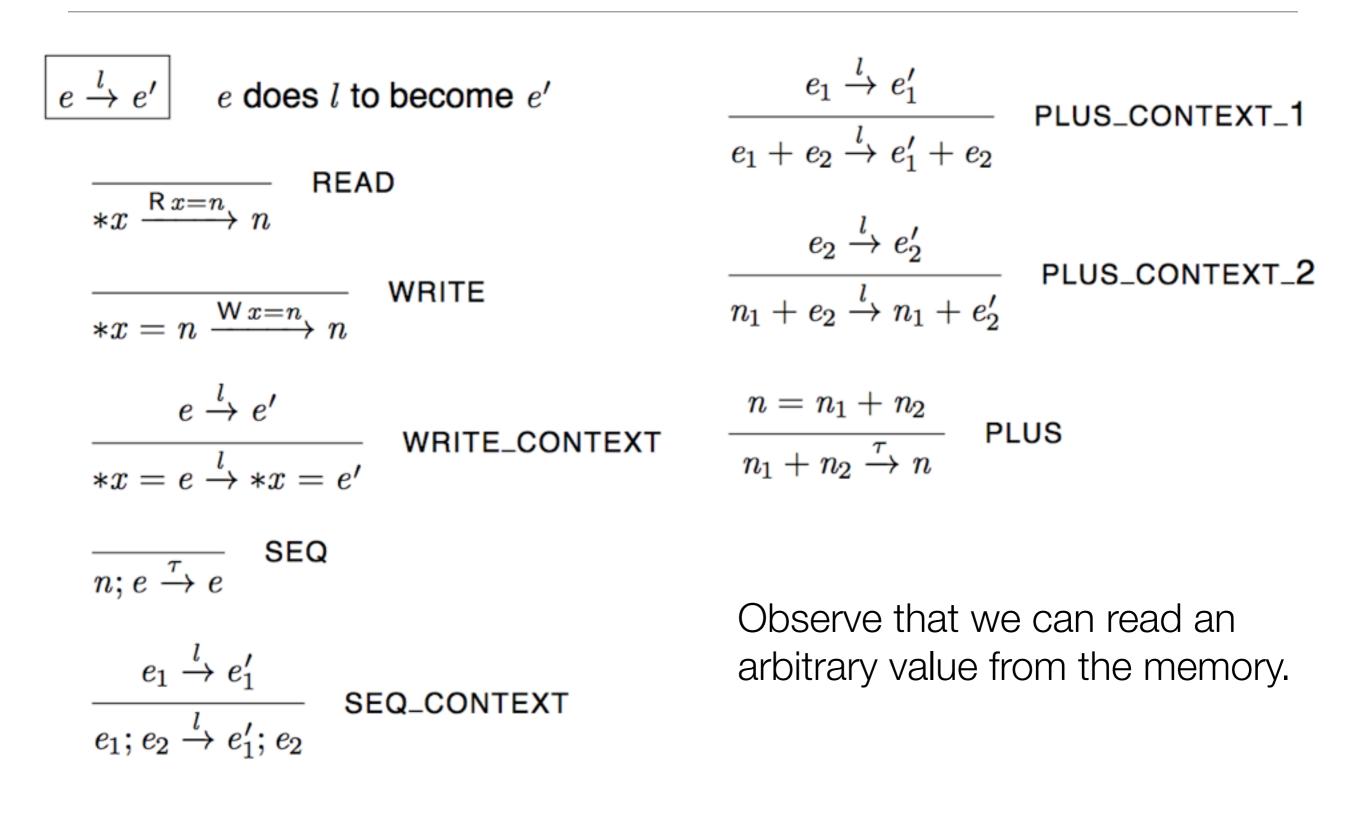
Separate language and memory semantics



A tiny language

location, x, m integer, n $thread_id, t$ k, i, j	address (or pointer value) integer thread id	
expression, e	$\begin{array}{cccc} ::= & & & \\ & & n & \\ & & *x & \\ & & *x = e \\ & & e; e' & \\ & & e + e' \end{array}$	expression integer literal read from pointer write to pointer sequential composition plus
process, p	$\begin{array}{c} ::= \\ & t : e \\ & p p' \end{array}$	process thread parallel composition

What can a thread do in isolation?



Example

Show that the expression:

$$(*x = *y); *x$$

can perform the following trace:

$$(*x = *y); *x \xrightarrow{\mathsf{R} y = 7} \xrightarrow{\mathsf{W} x = 7} \xrightarrow{\tau} \xrightarrow{\mathsf{R} x = 9} 9$$

Lifting to processes

$p \xrightarrow{l_t} p'$ p does	s l_t to become p'		
$\frac{e \xrightarrow{l} e'}{t: e \xrightarrow{l_t} t: e'} \text{THREAD}$		Actions are labelled by the thread that performed the action.	
$\frac{p_1 \xrightarrow{l_t} p_1'}{p_1 p_2 \xrightarrow{l_t} p_1' p_2}$	PAR_CONTEXT_LEFT	Free interleaving.	
$\frac{p_2 \xrightarrow{l_t} p_2'}{p_1 p_2 \xrightarrow{l_t} p_1 p_2'}$	PAR_CONTEXT_RIGHT		

A sequentially consistent memory

Take M to be a function from addresses to integers.

$$\begin{array}{l}
\underline{M \xrightarrow{l} M'} & M \text{ does } l \text{ to become } M' \\
\\
\underline{M(x) = n} \\
\underline{M \xrightarrow{Rx=n} M} & \text{MREAD} \\
\\
\underline{M \xrightarrow{Wx=n} M \oplus (x \mapsto n)} & \text{MWRITE}
\end{array}$$

SC semantics: whole system transitions

$$s \xrightarrow{l_t} s'$$
 s does l_t to become s'

Synchronising between the processes and the memory.

$$\frac{p \to p'}{\langle p, M \rangle \xrightarrow{\tau_t} \langle p', M \rangle} \quad \mathsf{STAU}$$

1

 τ_t

SC semantics, example

All threads read and write the shared memory. Threads execute asynchronously, the semantics allows any interleaving of the thread transitions.

$$\begin{array}{c} \langle t_{1}:*x = 1 | t_{2}:*x = 2, \ \{x \mapsto 0\} \rangle \\ \mathsf{W}_{t_{1}} x=1 \\ \langle t_{1}:1 | t_{2}:*x = 2, \ \{x \mapsto 1\} \rangle \\ \mathsf{W}_{t_{2}} x=2 \\ \langle t_{1}:1 | t_{2}:2, \ \{x \mapsto 2\} \rangle \\ \langle t_{1}:1 | t_{2}:2, \ \{x \mapsto 2\} \rangle \\ \langle t_{1}:1 | t_{2}:2, \ \{x \mapsto 2\} \rangle \\ \end{array}$$

Each interleaving has a linear order of reads and writes to memory.

...now we just have to define a TSO memory...

A sequentially consistent memory

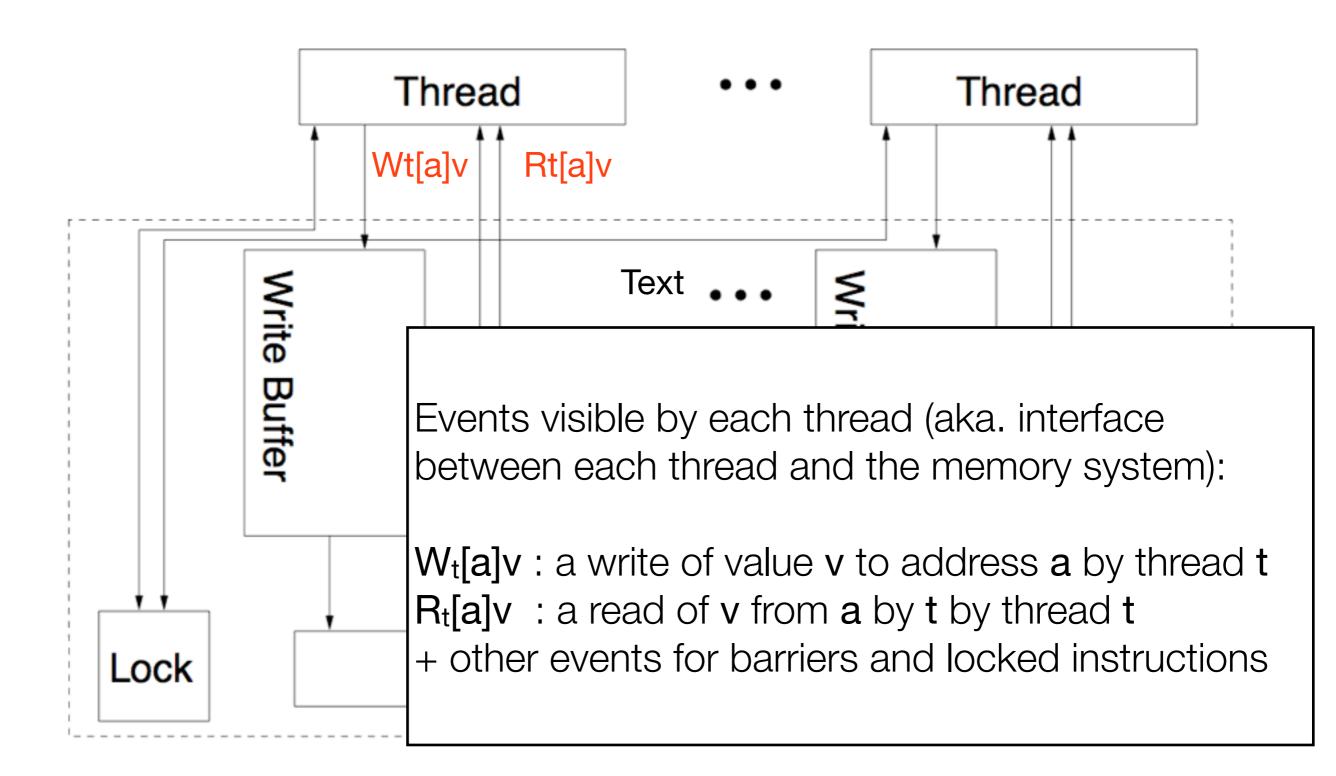
Take M to be a function from addresses to integers.

$$M \xrightarrow{l} M'$$
 M does l to become M'

$$\frac{M(x) = n}{M \xrightarrow{\mathsf{R} x = n} M} \quad \mathsf{MREA}$$

$$M \xrightarrow{\mathsf{W} x = n} M \oplus (x \mapsto n)$$

x86-TSO abstract machine



x86-TSO abstract machine

• The store buffers are FIFO. A reading thread must read its most recent buffered write, if there is one, to that address; otherwise reads are satisfied from shared memory.

• To execute a LOCK'd instruction, a thread must first obtain the global lock. At the end of the instruction, it flushes its store buffer and relinquishes the lock. While the lock is held by one thread, no other thread can read.

 A buffered write from a thread can propagate to the shared memory at any time except when some other thread holds the lock.

ues

x86-tso: a formalisation using an LTS

The machine state s can be represented by a tuple (M,B,L):

- M : address -> value option
- B : tid -> (address * value) list
- L : tid option

where:

M is the shared memory, mapping addresses to values

B gives the store buffer for each thread

L is the global machine lock indicating when a thread has exclusive access to memory (omitted in these slides)

x86-tso abstract machine: selected transition rules

t is *not blocked* in machine state s = (M,B,L) if [... or] the lock is not held.

In buffer B(t) there are *no pending writes* for address x if there are no (x,v) elements in B(t).

RM: Read from memory

not_blocked(s, t) s.M(x) = vno_pending(s.B(t), x) $s \xrightarrow{\mathsf{R}_t x = v} s$

Thread t can read v from memory at address x if t is not blocked, the memory does contain v at x, and there are no writes to x in t's store buffer.

x86-tso abstract machine: selected transition rules

RB: Read from write buffer not_blocked(s, t) $\exists b_1 \ b_2. \ s.B(t) = b_1 ++[(x, v)] ++b_2$ no_pending(b_1, x) $s \xrightarrow{R_t x = v} s$

Thread t can read v from its store buffer for address x if t is not blocked and has v as the newest write to x in its buffer;

x86-tso abstract machine: selected transition rules

WB: Write to write buffer

$$s \quad \xrightarrow{\mathsf{W}_t x = v} \quad s \oplus \langle\!\![B := s.B \oplus (t \mapsto ([(x,v)] + s.B(t)))]\!\!\rangle$$

Thread t can write v to its store buffer for address x at any time;

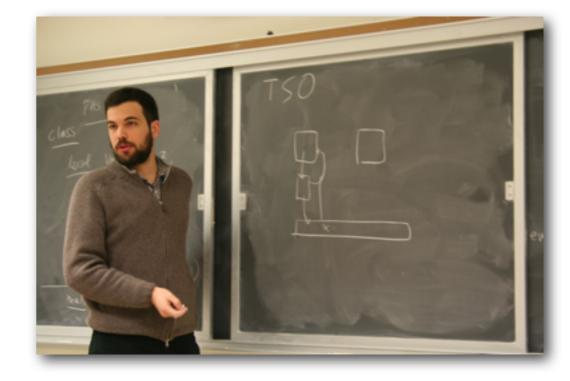
WM: Write from write buffer to memory

$$\operatorname{not_blocked}(s,t) \ s.B(t) = b ++[(x,v)]$$

 $s \xrightarrow{\tau_t x = v}$

 $s \oplus \langle\!\![M := s.M \oplus (x \mapsto v)]\!\!\rangle \oplus \langle\!\![B := s.B \oplus (t \mapsto b)]\!\!\rangle$

If t is not blocked, it can silently dequeue the oldest write from its store buffer and place the value in memory at the given address, without coordinating with any hardware thread

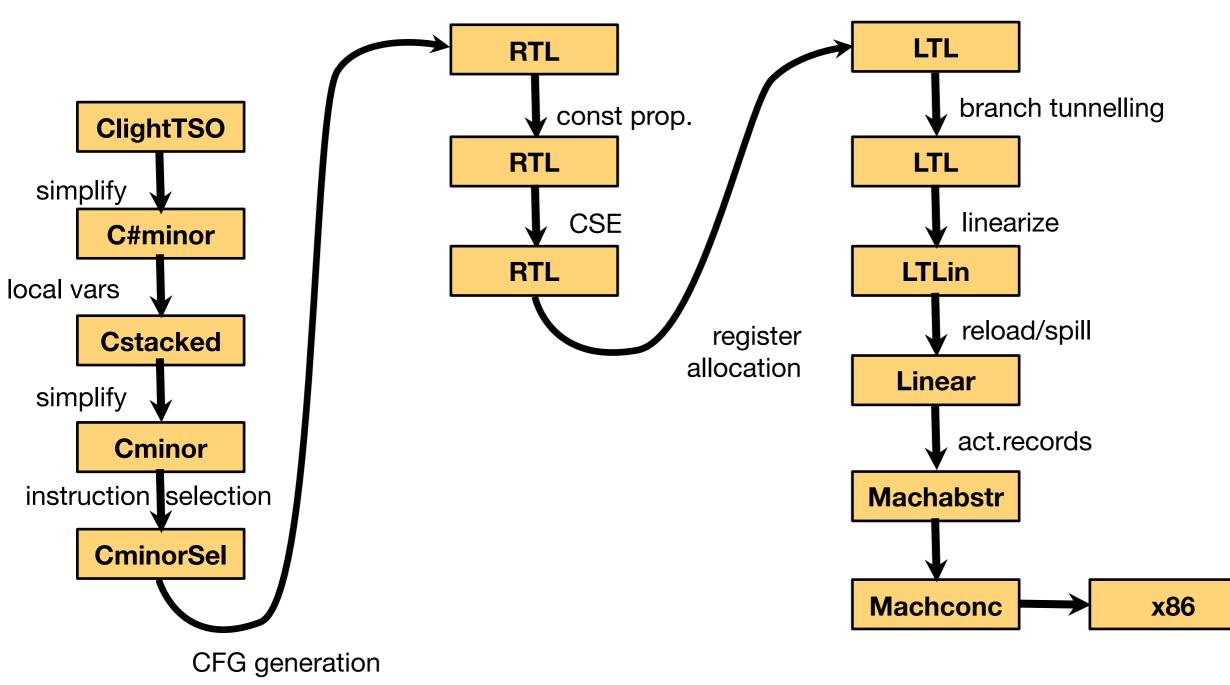


5. Veryfing fence elimination optimisations

aka reasoning on the x86TSO operational memory model and compiler correctness

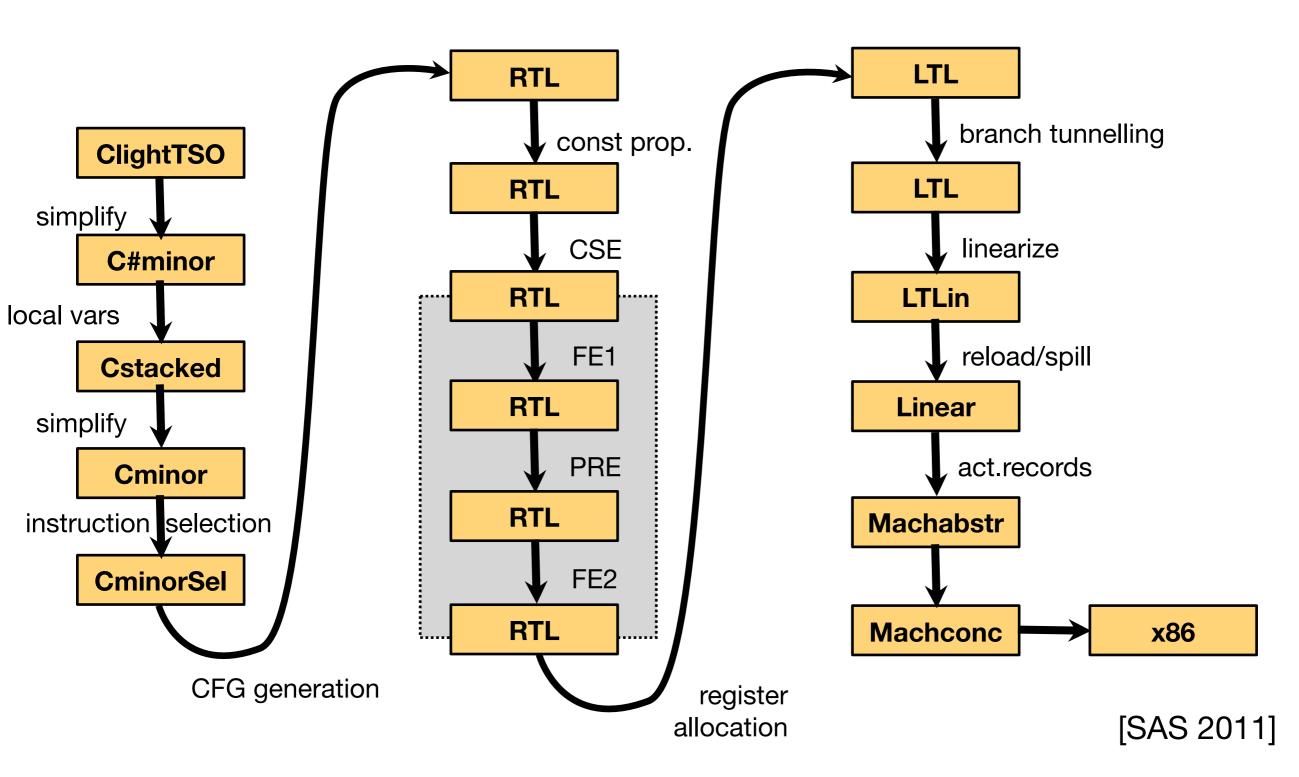


CompCertTSO

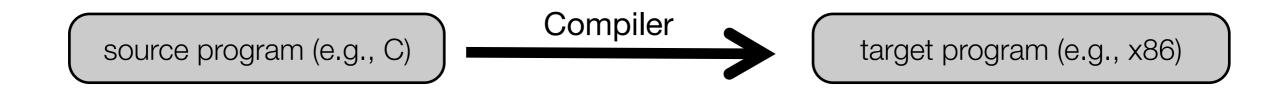


CompCertTSO + fence optimisations





Compilers are ideal for verification



Compilers are:

- Basic computing infrastructure
- Generally reliable, but nevertheless contain many bugs
 e.g., Yang et al. [PLDI 2011] found 79 gcc & 202 llvm bugs
- "Specifiable": compiler correctness = preservation of behaviours
- Interesting: naturally higher-order, involve clever algorithms
- Big, but modular

Language semantics

The semantics of all the CompCertTSO languages is defined by:

- a type of programs, prg
- a type of states, states
- a set of initial states for each program, init $\in prg \rightarrow \mathbb{P}(states)$
- a transition relation, $\rightarrow \in \mathbb{P}(\text{states} \times \text{event} \times \text{states})$

call, return, fail, oom, T

The visible behaviour of a program is defined by the external function calls (call) and returns (return), errors (fail), and running out of memory (oom).

Traces

- Finite sequences of call & return events ending with:

end: successful termination,

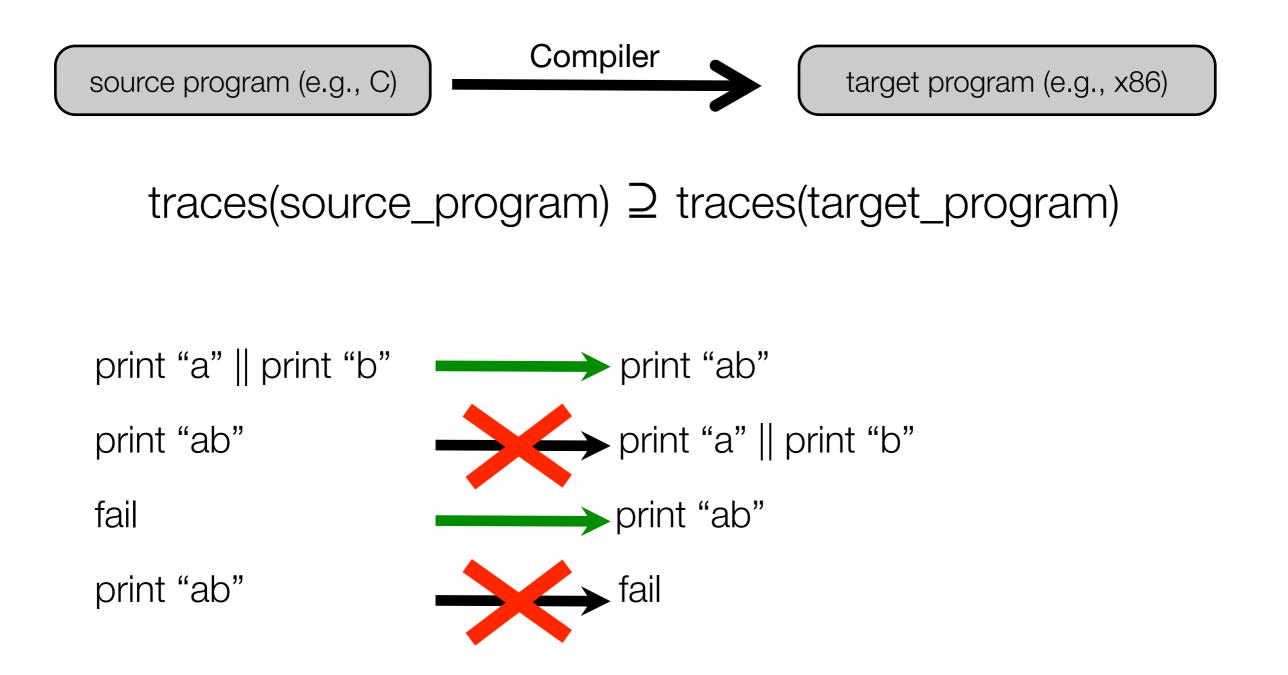
inftau: infinite execution that stops performing visible events oom: execution runs out of memory

- Infinite sequences of call & return events;

$$\begin{aligned} \operatorname{traces}(p) &\stackrel{\text{def}}{=} & \{\ell \cdot \operatorname{end} \mid \exists s \in \operatorname{init}(p). \ \exists s'. \ s \stackrel{\ell}{\Rightarrow} s' \wedge s' \not\rightarrow \} \\ & \cup \{\ell \cdot tr \mid \exists s \in \operatorname{init}(p). \ \exists s'. \ s \stackrel{\ell \cdot \operatorname{fail}}{\Longrightarrow} s'\} \\ & \cup \{\ell \cdot \operatorname{inftau} \mid \exists s \in \operatorname{init}(p). \ \exists s'. \ s \stackrel{\ell}{\Rightarrow} s' \wedge \operatorname{inftau}(s')\} \\ & \cup \{\ell \cdot \operatorname{oom} \mid \exists s \in \operatorname{init}(p). \ \exists s'. \ s \stackrel{\ell}{\Rightarrow} s'\} \\ & \cup \{tr \mid \exists s \in \operatorname{init}(p). \ s \text{ can do the infinite trace } tr\} \end{aligned}$$

NB: Erroneous computations become undefined after the first error.

Compiler correctness



Fence instructions prevent hardware reorderings

E.g., on x86-TSO:

[X]	=	[v]	=0	
L 1	JI			

Thread 0	Thread 1
MOV [x]←1	MOV [y]←1
MOV EAX←[y]	MOV EBX←[x]

Thread 0Thread 1
$$[x]=[y]=0$$
MOV $[x] \leftarrow 1$ MOV $[y] \leftarrow 1$ MFENCEMFENCEMOV EAX \leftarrow [y]MOV EBX \leftarrow [x]

EAX = EBX = 0forbidden

Who inserts fences?

1. The *programmer*, explicitly. Example: Fraser's lockfree-lib:

```
/*
 * II. Memory barriers.
 * MB(): All preceding memory accesses must commit before any later accesses.
 *
 * If the compiler does not observe these barriers (but any sane compiler
 * will!), then VOLATILE should be defined as 'volatile'.
 */
#define MB() __asm___volatile__ ("lock; addl $0,0(%%esp)" : : : "memory")
```

2. The *compiler*, to implement a high-level memory model, e.g. **SEQ_CST** C++0x low-level atomics on x86:

Load SEQ_CST: MFENCE; MOV Store SEQ_CST: MOV; MFENCE

Fence instructions

1. Fences are necessary

to implement locks & not fully-commutative linearizable objects (e.g., stacks, queues, sets, maps).

[Attiya et al., POPL 2011]

2. Fences can be expensive

If we have two consecutive fence instructions, we can remove the *latter*:



The *buffer* is already empty when the second fence is executed.

Generalisation:

MFENCE NON-WRITE INSTR ... NON-WRITE INSTR MFENCE MFENCE NON-WRITE INSTR ... NON-WRITE INSTR NOP

FE1

A fence is redundant if it always follows a previous fence or locked instruction in program order, and no memory store instructions are in between.

A forward data-flow problem over the boolean domain $\{\bot, \top\}$

Associate to each program point:

 ⊥ : along all execution paths there is an atomic instruction *before* the current program point, with no intervening writes;

op : otherwise.

 $T_1(nop, \mathcal{E})$ $= \mathcal{E}$ $= \mathcal{E}$ $T_1(\operatorname{op}(op, \vec{r}, r), \mathcal{E})$ $T_1(\texttt{load}(\kappa, addr, \vec{r}, r), \mathcal{E})$ $= \mathcal{E}$ $T_1(\texttt{store}(\kappa, addr, \vec{r}, src), \mathcal{E})$ = T $T_1(\texttt{call}(sig, ros, args, res), \mathcal{E}) = \top$ $T_1(\text{cond}(cond, args), \mathcal{E})$ $=\mathcal{E}$ $T_1(\text{return}(optarg), \mathcal{E})$ = T $T_1(\texttt{threadcreate}(optarg), \mathcal{E})$ = T $T_1(\texttt{atomic}(aop, \vec{r}, r), \mathcal{E})$ $= \bot$ $T_1(\texttt{fence}, \mathcal{E})$ = 1

 $\mathcal{FE}_{1}(n) = \begin{cases} \top & \text{if predecessors}(n) = \emptyset \\ \bigsqcup_{p \in \text{predecessors}(n)} T_{1}(\textit{instr}(p), \mathcal{FE}_{1}(p)) & \text{otherwise} \end{cases}$

A fence is redundant if it always follows a previous fence or locked instruction in program order, and no memory store instructions are in between.

FE1

A forward data-flow problem over $T_1(\texttt{nop},\mathcal{E})$ $= \mathcal{E}$ the bo \mathcal{E} Assoc Implementation: \perp : alc is a 1. Use CompCert implementation of Kildall algorithm CUr to solve the data-flow equations. no 2. Replace MFENCEs for which the analysis returns \perp = 1⊤:oth with NOP instructions. Ø $\mathcal{FE}_1(n)$ $\bigsqcup_{p \in \text{predecessors}(n)} T_1(\textit{instr}(p), \mathcal{FE}_1(p))$ otherwise

If we have two consecutive fence instructions, we can remove the *former*:



Intuition: the visible effects initially published by the former fence, are now published by the latter, and nobody can tell the difference.

Generalisation:

MFENCE INSTRUCTION 1 ... INSTRUCTION n MFENCE



NOP INSTRUCTION 1 ... INSTRUCTION n MFENCE

If there are reads in between the fences...

$$[x] = [y] = 0$$

$$Thread 0$$

$$MOV [x] \leftarrow 1$$

$$MOV [y] \leftarrow 1$$

$$MFENCE$$

$$MOV EAX \leftarrow [y]$$

$$MFENCE$$

$$MOV EBX \leftarrow [x]$$

$$EAX = EBX = 0$$
forbidden

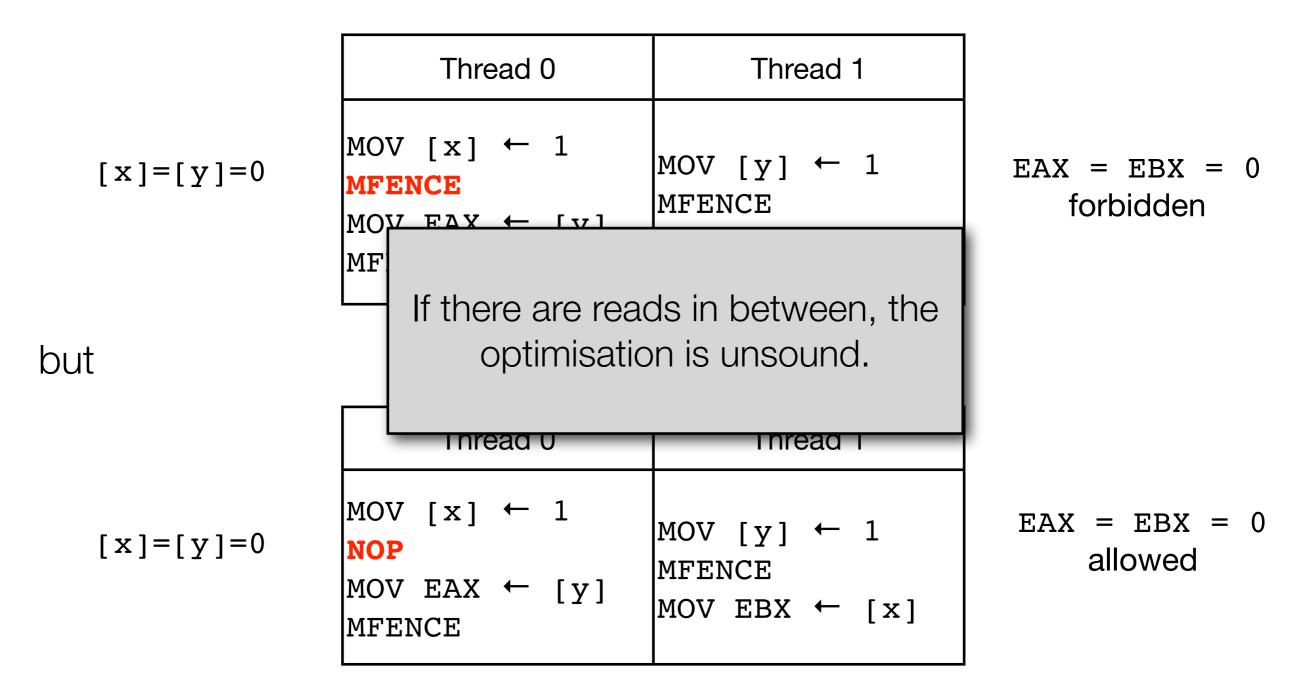
but

[x]=[y]=0

Thread 0Thread 1MOV
$$[x] \leftarrow 1$$
MOV $[y] \leftarrow 1$ NOPMOV EAX \leftarrow [y]MOV EAX \leftarrow [y]MFENCEMFENCEMOV EBX \leftarrow [x]

EAX = EBX = 0allowed

If there are reads in between the fences...



Swapping a **STORE** and a **MFENCE** is sound:

MFENCE; STORE



STORE; MFENCE

1. transformed program's behaviours \subseteq source program's behaviours (source program might leave pending write in its buffer)

2. There is the new intermediate state if the buffer was initially non-empty, but this intermediate state is not observable.

(a local read is needed to access the local buffer)

Intuition: Iterate this swapping...



A fence is redundant if it always precedes a later fence or locked instruction in program order, and no memory read instructions are in between.

A backward data-flow problem over the boolean domain $\{\bot, \top\}$

Associate to each program point:

 ⊥ : along all execution paths there is an atomic instruction *after* the current program point, with no intervening reads;

op : otherwise.

 $= \mathcal{E}$ $T_2(nop, \mathcal{E})$ $T_2(\operatorname{op}(op, \vec{r}, r), \mathcal{E})$ $= \mathcal{E}$ $T_2(\texttt{load}(\kappa, addr, \vec{r}, r), \mathcal{E})$ = T $T_2(\texttt{store}(\kappa, addr, \vec{r}, src), \mathcal{E})$ $=\mathcal{E}$ $T_2(\texttt{call}(sig, ros, args, res), \mathcal{E}) = \top$ $T_2(\text{cond}(cond, args), \mathcal{E})$ $=\mathcal{E}$ $T_2(\text{return}(optarg), \mathcal{E})$ = T $T_2(\texttt{threadcreate}(optarg), \mathcal{E})$ = T $T_2(\texttt{atomic}(aop, \vec{r}, r), \mathcal{E})$ $= \bot$ $T_2(\texttt{fence}, \mathcal{E})$ $= \bot$

$$\mathcal{FE}_{2}(n) = \begin{cases} \top & \text{if successors}(n) = \emptyset \\ \bigsqcup_{s \in \text{successors}(n)} T_{2}(\textit{instr}(s), \mathcal{FE}_{2}(s)) & \text{otherwise} \end{cases}$$

FE1 and FE2 are both useful

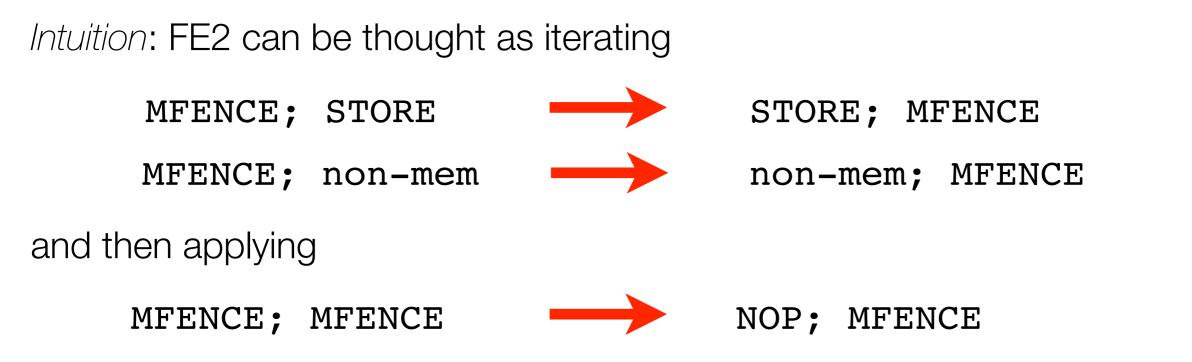
Removed by FE1 but not FE2:

MFENCE MOV EAX <- [y] MFENCE MOV EBX <- [y]

Removed by FE2 but not FE1:

MOV [x] <- 1 MFENCE MOV [x] <- 2 MFENCE

Informal correctness argument



This argument works for *finite traces*, but not for *infinite traces* as the later fence might never be executed:

MFENCE;
STORE;
WHILE(1);
MFENCE

NOP; STORE; WHILE(1); MFENCE

Basic simulations

Exhibiting a basic simulation implies: $traces(compile(p)) \setminus \{t \cdot inftau \mid t trace\} \subseteq traces(p)$ "simulation can stutter forever" **Definition 2 (Measured sim.).** A measured simulation is any basic simulation $(\sim, >)$ such that > is well-founded.

Theorem 1. If there exists a measured simulation for the compilation function compile, then for all programs p, traces(compile(p)) \subseteq traces(p).

Simulation for FE2

 $s =_i t$ iff thread *i* of *s* and *t* have identical pc, local states and buffers

 $s \sim_i s'$ iff thread *i* of *s* can execute zero or more NOP, OP, STORE and MFENCE instructions and end in the state *s*'

s ~ t iff

- -t's CFG is the optimised version of s's CFG; and
- s and t have identical memories; and
- \forall thread *i*, either $s \equiv_i t$ or

the analysis for *i*'s pc returned \perp and $\exists s', s \sim_i s'$ and $s' \equiv_i t$ "s is some instructions behind and can catch up"

Stutter condition:

t > t' iff $t \rightarrow t'$ by a thread executing a NOP, OP, STORE OF MFENCE (and t's buffer being non-empty)

Simulation for FE2

s ≡it iff th	read i of s and t have identical pc. local states and buffers					
s ∼i s' iff th Mfe:	But if (1) all threads have non-empty buffers, and (2) are stuck executing infinite loops, and (3) no writes are ever propagated to memory,					
s~t iff	then we can stutter forever.					
 <i>t</i>'s CFG <i>s</i> and <i>t</i> <i>t</i> thread 	(i.e., > is not well-founded.)					
the analysis for i's pc returned \perp and $\exists s', s \sim_i s'$ and $s' =_i t$						
	"s is some instructions behind and can catch up"					

Stutter condition:

t > t' iff $t \rightarrow t'$ by a thread executing a NOP, OP, STORE OF MFENCE (and t's buffer being non-empty)

Simulation for FE2

s ≡; t iff thr	read i of s and t have identical pc. local states and buffers					
s ∼i s' iff tł mfe:	(2) are stuck executing infinite loops, and					
s~t iff	then we can stutter forever.					
– t's CFG	(;					
 – s and t – ∀ thread 	(I. Solution 1: Assume this case never arises (fairness)					
Solution 2: Do a case split.						
	 If this case does not arise, we are done. 					
Stutter conc t > t' iff t	$t \rightarrow$ construct an infinite trace for the source					
t > t' iff t	 If this case does not arise, we are done. If it does, use a different (weaker) simulation to 					

Definition 3 (Weaktau sim.). A weaktau simulation consists of a basic simulation $(\sim, >)$ with and an additional relation between source and target states, $\simeq \in \mathbb{P}(src.states \times tgt.states)$ satisfying the following properties:

$$\begin{array}{ll} sim_weaken: \forall s, t. \ s \sim t \implies s \simeq t \\ sim_wstep: \forall s \, t \, t'. \ s \simeq t \wedge t \xrightarrow{\tau} t' \wedge t > t' \implies \\ & (s \xrightarrow{\tau} * \xrightarrow{\texttt{fail}} _) & -s \ reaches \ a \ failure \\ & \lor (\exists s'. \ s \xrightarrow{\tau} * \xrightarrow{\tau} s' \wedge s' \simeq t') & -s \ does \ a \ matching \ step \ sequence. \end{array}$$

Theorem 2. If there exists a weaktau-simulation $(\sim, >, \simeq)$ for the compilation function compile, then for all programs p, traces(compile(p)) \subseteq traces(p).

Remarks:

- Once the simulation game moves from ~ to \simeq , stuttering is forbidden;
- Can view difference between ~ and ~ as a boolean prophecy variable.

Weaktau simulation for FE2

 $s \sim t$, t > t' as before.

 $s \simeq t \text{ iff}$

- t's CFG is the optimised version of s's CFG; and

 $-\forall i, \exists S' \text{ s.t. } S \rightsquigarrow_i S' \equiv_i t.$

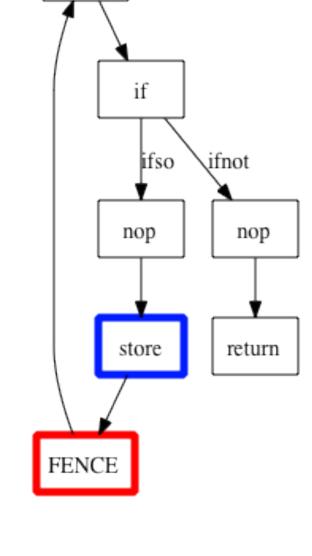
(i.e., same as $s \sim t$ except that the memories memories are unrelated.)

A closer look at the RTL

Patterns like that on the left are common.

FE1 and FE2 do not optimise these patterns.

It would be nice to hoist those fences out of the loop.

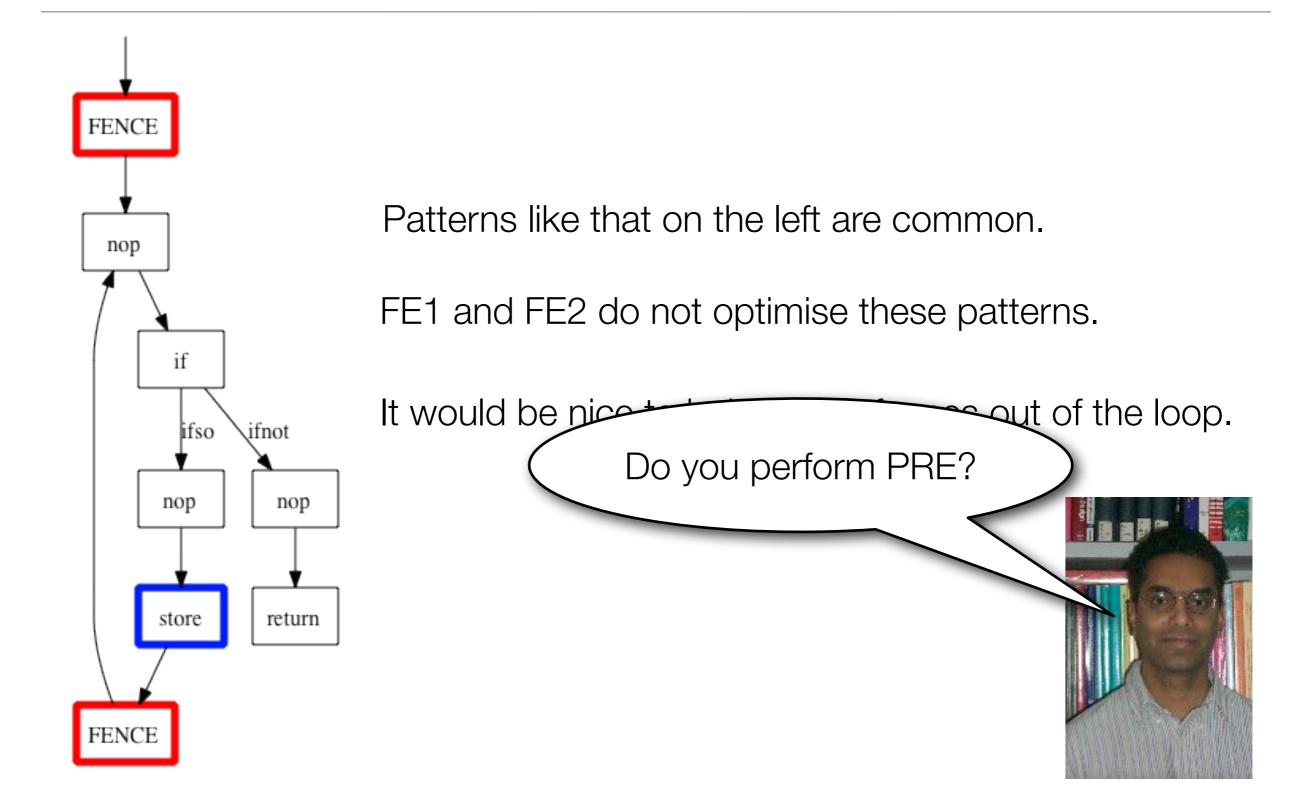


FENCE

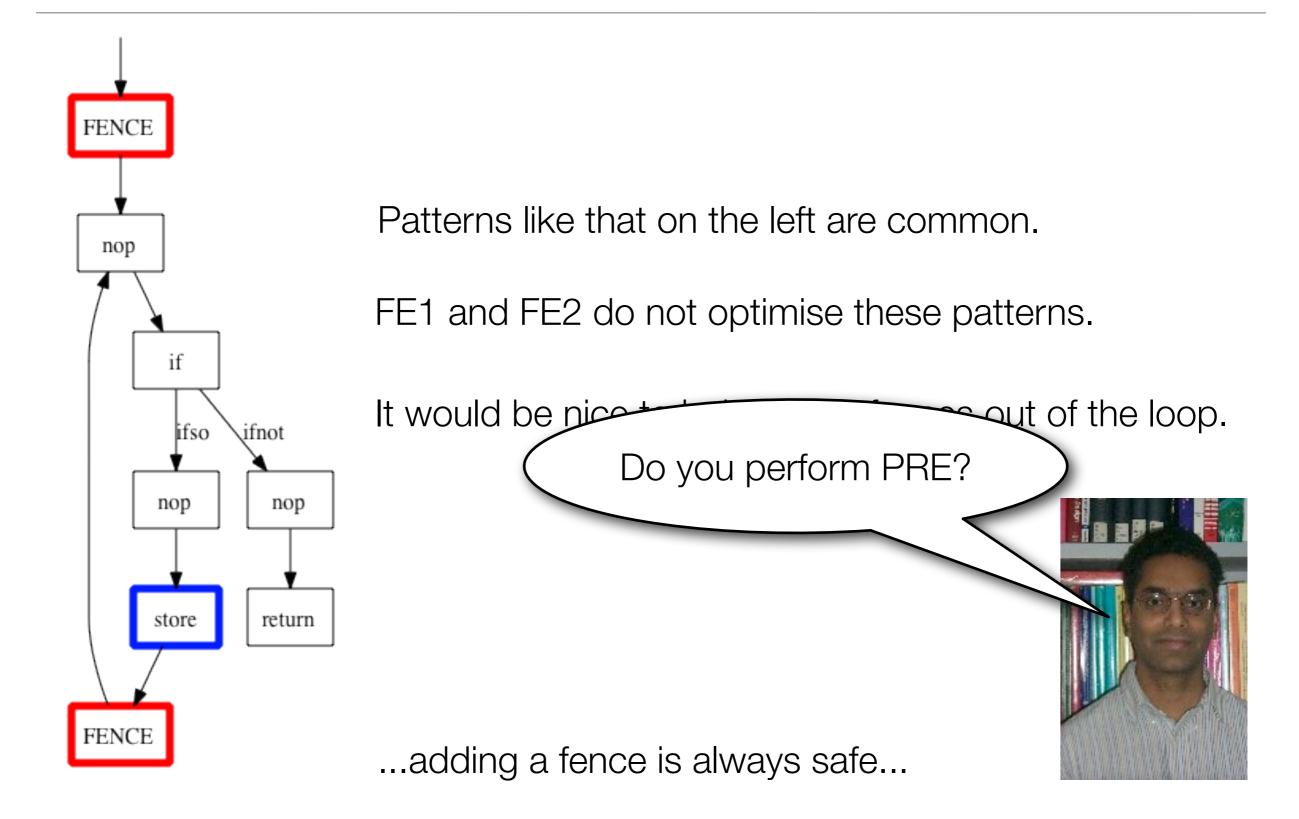
nop

Monday, 9 January 17

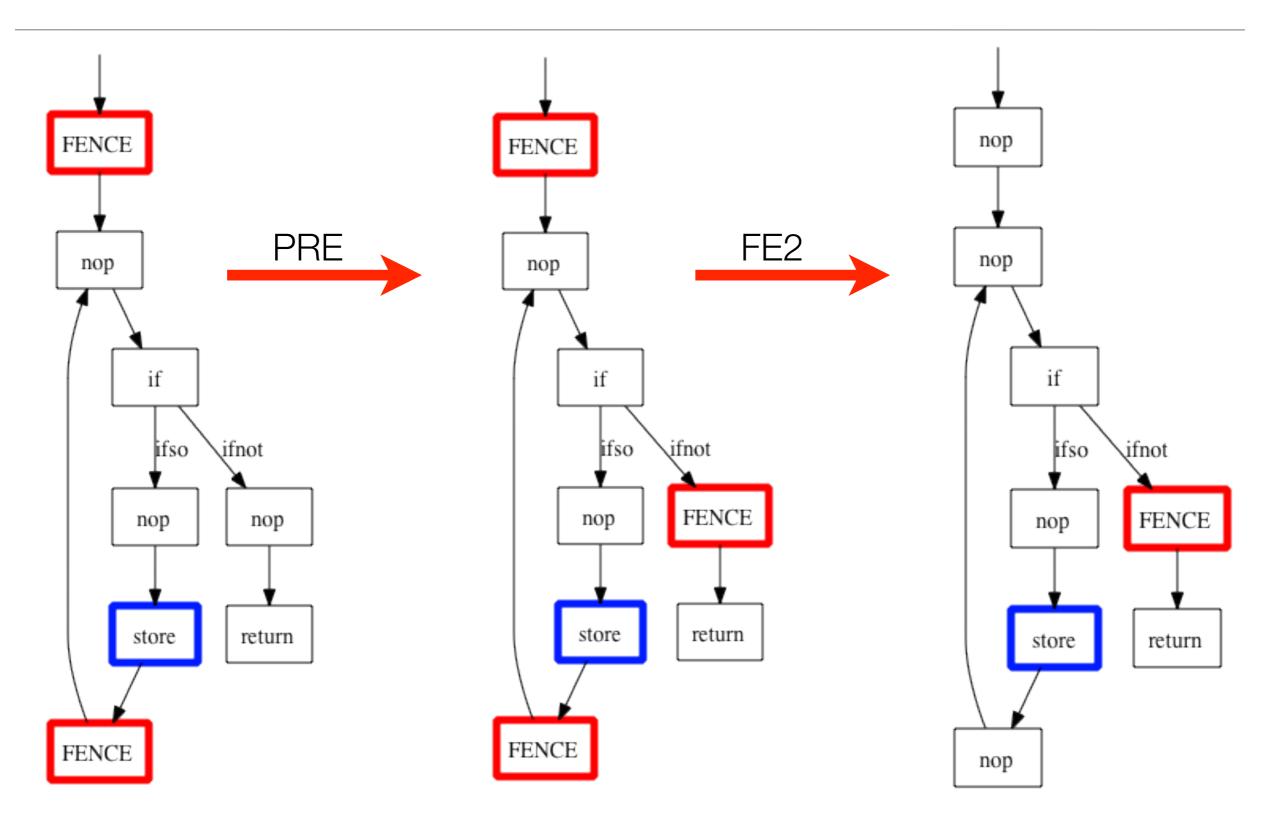
A closer look at the RTL



A closer look at the RTL



Partial redundancy elimination



Evaluation of the optimisations

- Insert MFENCEs before every read (br), or after every write (aw).
- Count the MFENCE instructions in the generated code.

	br	br+FE1	aw	aw+FE2	aw+PRE+FE2
Dekker	3	2	5	4	4
Bakery	10	2	4	3	3
Treiber	5	2	3	1	1
Fraser	32	18	19	12	11
TL2	166	95	101	68	68
Genome	133	79	62	41	41
Labyrinth	231	98	63	42	42
SSCA	1264	490	420	367	367

Evaluation of the optimisations

- Insert MFENCEs before every read (br), or after every write (aw).

